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DOCTORAL THESIS STATEMENT

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Department of Control Engineering

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Estimation of the stochastic properties of controlled systems

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Those interested may get acquainted with the doctoral thesis concerned at the Dean Office of the Faculty of Electrical Engineeringof the CTU in Prague, at the Department for Science and Research, Technická 2, Praha 6.

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Content

1. CURRENT SITUATION OF THE STUDIED PROBLEM

In the second half of the 20th century, a new state–space theory about dynamic systems was developed. The pioneering papers of the new theory were written by Rudolf E. Kalman, (Kalman 1960, 1961). The new theory considers not only input–output behavior as previously used transfer functions, but also the trajectories of the internal states. The state–space (SS) approach allows incorporating the first principles to the mathematical models. If not all of the states are measurable, they can be estimated by state observers. This allows using, for example, a state feedback even if some of the states are hidden.

In the 60‘s and 70's, a new theory was being developed remarkably quickly and it brought a new potential for solving various problems, especially in the field of control and regulation, (Kailath, 1979; Anderson and Moore, 1979; Kailath,*et al*. 2000; Gibbs, 2011). High computational power offers employing sophisticated control algorithms that can significantly improve the control performance. During past decades, the Model Predictive Control (MPC) algorithms has begun to be widely used for control of large processes such as petro chemical industry and power plants. If the model of a real system is known, it allows predicting the state and output trajectories which can be further used for the control optimization. To obtain the state space models, many identification methods have been developed, e.g. Subspace identification methods, Grey Box Modeling and others, (Ljung 1987; Katayama 2005; Řehoř, 2010, 2011). However, only minor attention was paid to identification of the stochastic part of the dynamic systems. Omitting uncertainties and noise properties can lead to significant limitation of the algorithms based purely on modeling of the deterministic part. As an example, consider a state estimator – Kalman filter. The quality of the state estimates depends on the knowledge of the deterministic part of the real system, but also on the knowledge of the stochastic properties and noise entering the process.

Throughout the thesis, linear dynamic systems affected by a Gaussian noise will be considered as the models of real processes. The corresponding state estimator for a linear system is given by the system matrices and the noise properties represented by covariance matrices.

The problem of finding a model of the deterministic part is solved by various identification methods. Examination and identification of the stochastic properties of real processes is studied in this thesis. If the real process is well identified, the deterministic as well as the stochastic part, then the predictions of such a model are accurate even for longer horizons. This leads to better estimation and prediction of the hidden system states and outputs. Accurate estimates/predictions can be used by controllers leading to better performance of the real process, less energy consumption, less pollution and increased safety which are the main goals of present technology.

Methods for identification of linear dynamic systems are being developed for over one century. At the beginning, input–output relation represented by transfer functions was used. In the 60's, the state space methods became popular and quickly improved. However, only minor attention was paid to the identification of the stochastic part of a system and noise properties.

In the 60's, the pioneering papers were written about the estimation of noise covariances, (Mehra 1970, 1972; Carew and Belanger 1973; Neethling and Young 1974; Belanger 1974). It was observed that the autocorrelation of the output prediction errors is linearly depended on the covariance matrices describing the entering Gaussian noise. Then, for several decades, this topic was quite overlooked by the control science. Some research was done within the fields of speech, acoustics, signal processing and statistics, but the knowledge was not sufficiently applied to solve the problems of the system and control theory.

The latest methods concerning the estimation of noise covariances were published in years 2005–2009, (Odelson, *et al*. 2005; Akesson, *et al*. 2008; Rajamani and Rawlings, 2009; Duník, *et al*. 2009). The main contribution of the recent papers was algorithms that offer a solution for finding the noise covariance matrices in the explicit form. The original paper was written by Odelson *et al*. and the further publications offer several extensions and modification of the original approach. The last mentioned reference, (Duník, *et al*. 2009), offers a comparison and discussion over the methods for the estimation of noise covariances.

Another approach is described in Pour *et al*. (2009) which describes a method for estimation of the covariance matrix using the innovation form of a linear system. This paper also proves the consistency of the estimation process. However, the initial Kalman gain is assumed to be a priori known for the given system, which simplifies the problem.

2. AIMS OF THE DOCTORAL THESIS

The goal of the thesis is developing new approaches and algorithms for analysis and identification of the stochastic part of dynamic systems. The main goal is to develop a Bayesian algorithm for estimation of the noise covariance matrices. For the analysis purposes, the models will be considered as discrete, linear and affected by Gaussian noise.

The main results of the thesis are separated into the individual chapters. The first main contribution of the thesis is covered in Chapter 3. It solves a question, whether the filter performance is optimal or not, i.e. whether the quality of the state estimates can be improved. The question is answered by examining the sequence of output prediction errors, which is the only measurable value. If the output prediction errors form a white sequence, than it holds, that the filter performance is optimal. Chapter 3, therefore, solves a problem if the given sequence is white or colored. Several different methods are described and compared to the widely cited method published by Mehra in 1970. The optimality tests are then used together with the noise covariance estimating algorithms as a form of adaptive Kalman filter.

The second part of the thesis contains a detailed description of the Bayesian approach used to estimate the covariance matrices of the entering noise from the given output data. Bayesian theory is used together with Monte Carlo numerical methods. Several modifications are discussed for this method, and an extensive comparison to the previously published algorithms is given.

The third part of the thesis provides an overview about the colored noise. In the typical application of the Kalman filter, it is assumed that the entering noise is white. However, this is not necessarily true, and neglecting the color property may lead to the poor state estimates. Chapter 5**Chyba! Nenalezen zdroj odkazů.** discusses how to detect whether the entering noise is colored or white using the time and the frequency domain. It also discusses whether it is possible to distinguish between colored process noise and colored measurement noise. The chapter further offers an overview of possible solutions. It also contains several numerical examples and highlights a significant potential of further research on this field.

The last part derives the Cramér–Rao bound for the estimation of the noise covariances. This bound represent the limitation of the estimation algorithms and can provide overview about possible convergence rates for any new approaches solving the estimation of the noise covariances. A numerical example demonstrates the performance of the Bayesian algorithm and the recent methods and compares the results to the Cramér–Rao bound.

Summary of the thesis goals

1) Analyze stochastic properties of linear dynamic systems.

2) Summarize, develop and compare algorithms for performance evaluation of a Kalman filter.

3) Develop an approach for estimation of the noise covariance matrices.

4) Analyze linear systems affected by colored noise. Develop a method for detection of a colored noise. Develop an approach that deals with the colored noise and demonstrate a potential of using a noise shaping filter.

5) Analyze the limits for quality of the noise covariance estimation algorithms using Cramér-Rao bounds.

3. WORKING METHODS

The main tool, for analysis of the stochastic of the stochastic properties of dynamic systems, was the statistics and probability theory. Bayesian approach was used to develop the algorithm for the estimation of noise covariance matrices. Bayessian approach often leads to non-linear and non-convex problems. To deal with these difficulties, we employed numerical methods called Monte Carlo. Monte Carlo represents a family of numerical methods that can deal with problems which cannot be solved analytically. The numerical algorithms can solve also high dimensional problems, however, the requirements for computer memory and power increase rapidly with the problem size. However, numerical simulasions represent a stratightforward way for solving hard problems.

Throughout the thesis, a linear dynamic system was employed for the analysis. Identification and analysis of the stochastic properties was the main goal. More concretly, the noise covariance matrices were of the interest. The paper Odelson *at al*. (2005) describes an algebraic method for identification of the noise covariance matrices. In the doctoral thesis, the Bayesian approach was applied. This approach develops probability distribution functions rather than simple point estimates. The probability distribution functions provides significantly more information about the estimation quality. Statistical approach provides also a quality measure called Cramér-Rao bounds, which allows to define the optimal performance and limits for the performance quality. We have derived the Cramér-Rao bounds for the problem of estimating the noise covariances and the proposed alogorithm was compared to the Cramér-Rao bounds.

Chapter 5 of the thesis deals with detection of colored noise. Two approaches were used - the time and the frequency domain analysis. Statistical methods were used in the time time domain, and spectral densities in the frequency domain. Thi chapter employs a polynomial methods to demonstrate equivavelncy of two different noise models.

4. RESULTS

The doctoral thesis covered a large part of the stochastic properties estimation for linear dynamic systems. First part discussed optimality tests, that can be used for evaluation of the state estimation quality using the innovation sequence. The whiteness tests are widely used by statisticians, however, they can be hardly found in the control field literature. Technologists and scientists who use the Kalman filter for state predictions usually do not use any quality measure. We have chosen the most significant whiteness property tests and demonstrate their performance with various systems. For this purpose we analyzed and compared several estimators for the autocorrelation function. We have proposed an estimator that generates independently identically distributed values. Further, a quality measure has been proposed together with the decision about optimality. The quality measure can be used to compare the settings of a Kalman filter that are available. It can be also used to detect changes in the noise intensity or structure.

The second part of the thesis described algorithms for the noise covariance estimation. It started with demonstration that the Bayesian principles can extract enough information from the output data for the estimation of noise covariances. Together with Monte Carlo methods, we have proposed an algorithm that is better than previously published methods and works also with MIMO systems. Combining the quality measures and the estimation algorithm we have proposed an adaptive Kalman filter, that can use a newly measured data to update the covariances and evaluate the filter performance.

The third part of the thesis used the knowledge about the noise covariance estimation and the optimality tests. We have shown how to detect that the entering noise is colored. The time and the frequency domain analyses were employed to decide whether the given system is able to generate the output spectral density the same as the measured signal has. We also introduced a simple practical method for finding the shaping filter. It was demonstrated on a numerical example, how the Kalman filter performance can be improved when the appropriate shaping filter is used and the color property of the entering noise is not neglected. We pointed out the spectrum of the innovation sequence and also the state prediction error.

The last part discussed the limits of the estimation quality for the noise covariances. We have employed the Cramér–Rao bounds and calculate the bounds for this particular problem using the approach proposed by Šimandl*et al*. (2006). We have shown some interesting properties of the Cramér–Rao bounds for this problem. The fact that we have calculated the Cramér–Rao bounds only for a scalar system does not mean any significant disadvantage. Any newly proposed algorithm can be compared to the Cramér–Rao bounds employing the scalar system. It can be expected that if the performance for small system is far from the optimum, the algorithm would not work well for larger systems.

Several of the functions used for calculations throughout the thesis are attached in Appendix A in m–code for Matlab. Most of them are necessary for ALS/scALS method, however, they are ready for a general use.

5. CONCLUSION

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The doctoral thesis meets all its assignments and goals. The original results have been presented at high impact conferences and journals. The results presented here can be used for practical applications and also as an inspiration for a further research in the field of stochastic system identification.

6. LIST OF LITERATURE USED IN THE THESIS STATEMENT

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7. LIST OF CANDIDATE'S WORKS RELATING TO THE DOCTORAL THESIS

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8. SUMMARY

The doctoral thesis covers a part of the stochastic properties identification for linear dynamic systems. Knowledge of the noise sources and uncertainties is essential for the state estimation performance. The covariances of the process and measurement noise represent tuning parameters for the Kalman filter and the state estimation quality depends directly on them. The thesis deals with estimation of the noise covariances from the measured data. A Bayesian approach together with Monte Carlo methods are employed for this task. The thesis describes optimality tests that can be used to evaluate the quality of the state estimates obtained by a Kalman filter. A new approach was introduced to detect the color property of the process noise. If the process noise is colored, the shaping filter can be found in the time or frequency domain. It can be added to the Kalman filter which can be then tuned optimally. The limitations of the noise covariance estimation are evaluated by the Cramér–Rao bounds. The convergence of the proposed algorithms and the previously published ones were compared.

9. RÉSUMÉ

Tato disertační práce se zabývá identifikací stochastických vlastností lineárních dynamických systémů. Znalost stochastických vlastností a neurčitostí tvoří hlavní podmínku pro kvalitu odhadování a filtrace. Vlastnosti šumu, jako bělost a kovarianční matice, vstupují do syntézy pozorovatelů stavů jako ladící parametry. Jejich znalost přímo ovlivňuje kvalitu odhadu a predicki stavu. Pokud predikce a filtrace poskytují hodnoty blízké skutečným stavům, je možné pozorovatele využít například pro prediktivní řízení, které postupně získává stále víc průmyslových aplikací.

Disertační práce si klade za cíl analýzu stochastických vlastností lineárních procesů a návrh algoritmů pro jejich odhad. V práci se široce využívá Bayesovská teorie spolu s numerickými metodami Monte Carlo. Hlavním přínosem práce je návrh algoritmů pro odhad kovariančních matic šumu vstupujícího do procesu. Spolu s odhadovacími algoritmy byly navrženy i testy optimality, ktoré poskytují kvalitativní hodnocení odhadů stavů, které generují pozorovatele.

Práce se dále zabýva detekcí barevného šumu, který je v teorii Kalmanské filtrace často zanedbáván. Ukázali jsme, jak lze detekovat barevný šum pomocí analýzy v časové i frekvenční oblasti. Načrtli jsme způsob, jak najít odpovídající tvarovací filtr šumu a jak jej implementovat do Kalmanova filtru. Zároveň jsme poukázali na nejednoznačnosti, které přináší pozorování výstupních dat.

V poslední části práce jsme odvodili Cramérovy-Raovy meze, které představují limity pro kvalitu odhadu. Ukázali jsme několik zajímavých souvislostí mezi odhadem stavu a odhadem kovariančních matic. Demonstrovali jsme souvislost mezi Riccatiho rovnicí a Cramérovymi-Raovymí mezemi. Meze kvality odhadu jsme využili pro hodnocení kvality navrženého algoritmu pro odhad kovariančních matic a porovnali jsme nový algoritmus s dříve publikovanými.