Czech Technical University in Prague
Faculty of Electrical Engineering

BACHELOR THESIS

Traffic Simulator for Highway Tunnels

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Název tématu: Simulátor dopravy v dálničních tunelech

Zásady pro vypracování:

1. Seznamte se s metodami algoritmizace pohybu silničních vozidel.
2. Vyberte metodu, která je vhodná pro simulaci pohybu vozidel v dálničním tunelu.
3. Metodu implementujte a vhodným způsobem graficky demonstrujte její funkci.

Seznam odborné literatury:

- Rothery, R. W. "Car Following Models," In: Rathi, A. K. (Ed.), Traffic Flow Theory: A State of the Art Report, FWHA, 2002, New York.
- Ferkl, L., Kurka, L., Sládek, O., and Pořízek, J., 2005. "Simulation of traffic, ventilation and exhaust in a complex road tunnel," In Proc. of IFAC 2005, Prague.

Vedoucí bakalářské práce: Ing. Lukáš Ferkl

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V Praze, dne 6. 3. 2007

Prohlášení

Prohlašuji, že jsem svou bakalářskou práci vypracoval samostatně a použil jsem pouze podklady (literaturu, projekty, SW atd.) uvedené v přiloženém seznamu.

Souhlasím s užitím tohoto školního díla ve smyslu § 60 Zákona č.121/2000 Sb. , o právu autorském, o právech souvisejících s právem autorským a o změně některých zákonů (autorský zákon).

V Praze dne ______________   ____________________
podpis

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Abstract

This thesis describes the best–known and most used algorithms for simulation of highway traffic. It describes pros and cons of each of the variety of methods and tries to choose the best–suited one for proper implementation of traffic simulator in a highway tunnel. The chosen method is developed into a suitable form and implemented by usage of C# language for .NET platform. The developed programme, Traffic Simulator, has user-friendly environment with possibility to model an arbitrary highway made of user–blocks and simulate the behaviour of cars according to the chosen method, and, with possibility of tunnel–ventilation and air–flow modeling in future.

Anotace

Tato práce popisuje nejznámější a nejpoužívanější algoritmy pro simulaci silniční dopravy. Popisuje výhody a nevýhody každé z použitých metod a pokouší se vybrat nejvhodnější pro správnou implementaci řízení dopravy v silničním tunelu. Vybraná metoda je rozvinutá do vhodné formy a implementovaná v jazyce C# platformy .NET. Vyvinutý program, Trenažér Tunelu, má přátelské prostředí s možností modelování libovolné dálnice a do budoucna je připraven k rozšíření o modelování proudění vzduchu.

Contents

Notations

Variable	Description	Unit
$\alpha_f(t)$	Instantaneous acceleration of a following vehicle at time t	ms^{-2}
$\alpha_l(t)$	Instantaneous acceleration of a lead vehicle at time t	ms^{-2}
δ	Short, finite time period	s^{-1}
k	Traffic stream density in vehicles per meter	m^{-1}
λ	Proportionality factor	$-$
q	Flow in vehicles per hour	h^{-1}
t	Time	s
T	Reaction time	s
U_l	Speed of a lead vehicle	ms^{-1}
U_f	Speed of a following vehicle	ms^{-1}
$\ddot{x}_f(t)$	Instantaneous acceleration of a following vehicle at time t	ms^{-2}
$\ddot{x}_l(t)$	Instantaneous acceleration of a lead vehicle at time t	ms^{-2}
$\dot{x}_f(t)$	Instantaneous speed of a following vehicle at time t	ms^{-1}
$\dot{x}_l(t)$	Instantaneous speed of a lead vehicle at time t	ms^{-1}
$x_f(t)$	Instantaneous position of a following vehicle at time t	m
$x_l(t)$	Instantaneous position of a lead vehicle at time t	m
$x_i(t)$	Instantaneous position of a i-th vehicle at time t	m
$g(x,t)$	Generation (dissipation) rate of vehicles	s^{-1}
α	Sensitivity coefficient descibing the intensity of interactions	s^{-1}
τ	Interaction time lag	s
$k(x,t)$	Density of the i-th lane	m^{-1}
k_i	Equilibrium density of the i-th lane	m^{-1}

Chapter 1

Introduction

1.1 State of the Art

The thesis gets to know with variety of traffic control and simulation approaches and methods, from the classic (Rothery, R.W. and Rathi, A.K., 2002) to most up-to-date methods (Ferkl, L., 2007), (Lee, H.K. et al., 2001) or (Knospe, W. et al., 2002). Traffic flows and air pollution are linked together in many different ways. It must be said, that most of the traffic simulations are not made for themselves, but for understanding and decreasing air poluttion. The time dimension for traffic management and control to reduce air pollution can range from short to medium to long; from days, to years, to decades; from almost immediate reactions to incidents, to pre–emptive measures, to long term strategies. For modelling and simulation is medium-term planning the most ineteresting one, because the short–term is difficult to compute and verify and long–term is difficult to reproduce. Taken these into account it is required to develop both the transport and the complex air pollution simulator.

It is not a new idea to develop some kind of traffic simulator; there has already been several trials to do so. Nevertheless all of those tried to implement the most complex thus most demanding model. Most of them are able to simulate minutes, hours, or days in maximum for high cost – the simulation takes days, weeks and in some cases months. These approaches try to simulate vast and complex areas of city district with thousands of cars and tens of streets. It means that there is no chance for real–time simulation although on the other hand some of their results might be useful. Probably the best–known of the above mentioned traffic simulators is *SIMTRAP*. It works typically with areas up to 200 km, which requires considerable computational resources. In most cases, typical end users (e.g., transportation planners) do not have access to such high–performance computing equipment.

1.2 Objectives of the Thesis

One of the main goals of the thesis is a developement of user-friendly and easy-to-use programme which will be able to simulate traffic in real–time and thus provide a preview of what will happen in some cases in the future by enabling simulation of days and weeks in minutes or hours and still with satisfying accuracy. Therefore, the chosen method must be simple enough to enable real–time simulation, but powerfull and accurate enough to provide reliable and useful data. It must be taken into account that this programme is primarily designed for highway tunnels, although it may be used for normal highway traffic, however, not for complex city districts or cities themselves. This thesis should prepare a simulator of traffic with ability to upgrade to complex traffic and air pollution simulator.

1.3 Outline of the Thesis

- **Chapter 1 – Introduction** presents the topic and goals of the thesis.
- **Chapter 2 – Traffic** describes variety of options in traffic modelling and compares them with each other. In this chapter the choice of the method best–suited for implementation of Traffic Simulator is described.
- **Chapter 3 – Implementation** descibes thr discretization of the continuous method chosen in the previous chapter. It also presents the algorithm used for implementation of the Traffic Simulator.
- **Chapter 4 – Conclusion** makes a summary of the goals described in this thesis and proposes future development of the Traffic Simulator.

Chapter 2

Traffic

2.1 Traffic Models

This chapter introduces various models that have been developed to describe traffic. During the last century, four basic approaches of mathematical decomposition of the traffic models were developed:

- Network model
- Macroscopic model [1]
- Car-following model [2]
- Continuum flow model

Basic versions of all of the above mentioned methods present a pure statistical approach, which can be satisfactory while only static dependencies are needed as the required data are measured in long-term runs. For more accurate model the traffic might not be presented only statistically (Ferkl, L., 2007); it is inevitable to come up with dynamics. Network model theory is briefly mentioned in (Ferkl, L., 2007) or (Kutil, M. et al., 2006), macroscopic and car-following models are described in a report published by the Federal Highway Administration (FHWA), edited by Rathi (Williams, C.J., 2002, chapter: Macroscopic model), (Rothery, R.W. and Rathi, A.K., 2002, chapter:

[1]There is a little discrepency in literature. Some authors (Rothery, R.W. and Rathi, A.K., 2002) use term macroscopic flow model, others (Ferkl, L., 2007) just the macroscopic model

[2]The similar case as with above. Some authors use term car-following model (Rothery, R.W. and Rathi, A.K., 2002) or (Brackstone, M. and McDonald, M., 2000), others use term microscopic model (Ferkl, L., 2007)

Car-following models) and continuum-flow model is described by Kuhne and Michalopoulos (KUHNE, R. and MICHALOPOULOS, P., 2002).

The network model uses Petri nets and it is convenient to use it for small scale traffic scenarios. Macroscopic model was developed with rapid urban growth and the demand for complex city traffic. The flow theory had to be extended to netwotk level. The traffic system consists of the network topology (street width and configuration) and the traffic control system (e.g., traffic signals, designation of one and two–way streets, and lane configuration). Because of the purpose of these models, they are out of scope of this thesis.

2.1.1 Car-Following Model

This approach was described in detail in (ROTHERY, R.W. and RATHI, A.K., 2002). There are several subtasks involved in the overall driving task such as perception, decision making, control and many others. This model was based upon one of these subtasks, the ability of driver to follow the car before. This subtask was picked up for this model with respect to its simplicity comparing to the other driving tasks and it has also been succesfully described by mathematical models.

The main idea of this approach is to model every single vehicle. Car-following model of a single lane assumes that each driver in following vehicle is an active and predictable control element in the system. There has been introduced stimulus-response equation that expresses the concept of driver's response to given stimulus:

$$\text{Response}{=}\lambda \text{ Stimulus} \tag{2.1}$$

where λ is a proportinality factor which equates the stimulus function to the response or control function, which of course can be composed of many factors: speed, relative speed, acceleration, driver's thresholds, etc. There is a time threshold (approximately 0.5 sec) (ROTHERY, R.W. and RATHI, A.K., 2002) for which a driver cannot evaluate the information given to him. One approach is to assume that

$$\sigma(t) = \delta(t - T) \tag{2.2}$$

where

$$\delta(t - T) = 0, \text{for } t \neq T \tag{2.3}$$

$$\delta(t - T) = 1, \text{for } t = T \tag{2.4}$$

and

$$\int_0^1 \delta(t-T)dt = 1$$

where $\sigma(t)$ is a weighting function which reflects driver's estimation, evaluation, and processing of earlier information (Chandler, F.E. et al., 1958), and $\delta(t)$ is time period. In this case, Stimulus function becomes

$$\text{Stimulus } (t) = U_l(t-T) - U_f(t-T) \tag{2.5}$$

where U_l is speed of the lead vehicle of a platoon and U_f is speed of the following vehicle. The driver is observing the Stimulus and determing a response that will be made some time in the future. By delaying the response, the driver obtains an "advanced" information.

The response function is taken as the acceleration of the following vehicle, because driver has direct control of this quantity through accelerator and break pedals and also because driver obtains direct feedback of this variable through inertial forces, i.e.,

$$\text{Response } (t) = a_f(t) = \ddot{x}_f(t) \tag{2.6}$$

where x_i denotes the longitudinal position along the roadway of the i-th vehicle at time t. Combining Equations (2.5) and (2.6), the stimulus-response equation becomes

$$\ddot{x}_f(t) = \lambda\left[\dot{x}_l(t-T) - \dot{x}_f(t-T)\right], \tag{2.7}$$

or equivalently

$$\ddot{x}_f(t+T) = \lambda\left[\dot{x}_l(t) - \dot{x}_f(t)\right]. \tag{2.8}$$

Equation (2.8) is an approximation of the stimulus-response equation of the car-following model. A generalisation of the car-following model in a conventional control theory block diagram is shown in Fig. 2.1.

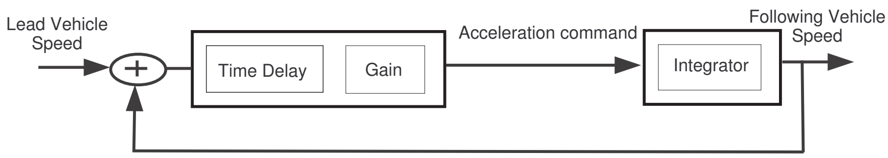

Figure 2.1: Block Diagram of the Linear Car-Following Model.

2.1.2 Continuum Flow Model

This model was developed as an analogy to fluid flow. Therefore there are often used terms such as flow, concentration and speed.

The theory of fluid dynamics is based upon Euler equations of continuity (valid for incompressible fluids). They correspond to Navier-Stokes equations with zero viscosity and heat conduction terms (compressible fluids).

Definition 2.1 (Del Operator): In the three-dimensional Cartesian coordinate system R^3 with coordinates (x, y, z), del operator is defined as

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

where (**i**,**j**,**k**) is the standard basis in R^3.

Note: Del is a vector differential operator represented by the nabla (∇) symbol. Mathematically, del can be viewed as the derivative in multi-dimensional space. When used in one dimension, it takes the form of the standard derivative of calculus. □

Definition 2.2 (Divergence): Let x, y, z be a system of Cartesian coordinates on a 3-dimensional Euclidean space, and let **i**, **j**, **k** be the corresponding basis of unit vectors. The divergence of a continuously differentiable vector field $\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$ is defined to be the scalar-valued function:

$$div\mathbf{F} = \nabla\mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Note: In vector calculus, the divergence is an operator that measures a vector field's tendency to originate from or converge upon a given point. For instance, for a vector field that denotes the velocity of air expanding as it is heated, the divergence of the velocity field would have a positive value because the air is expanding. Conversely, if the air is cooling and contracting, the divergence would be negative. □

Theorem 2.1 (Euler equation of continuity–conservation of mass):

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \boldsymbol{u}) = 0.$$

Proof: Suppose we have a fluid with local density $\rho(t, x, y, x)$ and local velocity $\mathbf{u}(t, x, y, z)$. Consider a control volume V (not necessarily small, not necessarily rectangular) which has boundary S. The total mass in this volume is

$$M = \int_V \rho dV. \tag{2.9}$$

The rate–of–change of this mass is just

$$\frac{\partial M}{\partial t} = \int_V \frac{\partial \rho}{\partial t} dV. \tag{2.10}$$

The only way such change can occur is by stuff matter flowing across the boundary, so

$$\frac{\partial M}{\partial t} = \int_S \rho \mathbf{u} d\mathbf{S}. \tag{2.11}$$

We can change the surface integral into a volume integral using Green's[3] theorem, to obtain

$$\frac{\partial M}{\partial t} = -\int_V \nabla(\rho \mathbf{u}) dV. \tag{2.12}$$

Comparing Equation (2.10) with Equation (2.12) we can see that they are equal no matter what volume V we choose, so the integrands must be pointwise equal. Therefore these equations can be rewritten as

$$\frac{\partial}{\partial t} \int_V \rho dV + \int_V \nabla(\rho \mathbf{u}) dV = 0. \tag{2.13}$$

The expression above is valid for V, which is a control volume that remains fixed in space. Because V is invariant in time, it is possible to swap the $\frac{\partial}{\partial t}$ and $\int_V dV$ operators. And as the expression is valid for all domains, we can additionally drop the integral. This gives us an expression for the local conservation of mass (the well-known **Euler equation of continuity**):

$$\frac{\partial \rho}{\partial t} + (\nabla \rho \mathbf{u}) = 0, \tag{2.14}$$

or equivalently as

$$\frac{\partial \rho}{\partial t} + div(\rho \mathbf{u}) = 0. \tag{2.15}$$

By transforming Equation (2.15) from R^3 to R we obtain

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0. \tag{2.16}$$

By re–denoting ρ as k, with

$$q = ku$$

and relationship between the mean speed u and the traffic density under equilibrium conditions

$$u = u_e(k), \tag{2.17}$$

[3]The Green's theorem is described on (Wikipedia, 2007) in detail.

we obtain

$$\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = 0. \tag{2.18}$$

Equation (2.18) expresses the law of conservation of a traffic stream and is known as the conservation or continuity equation. If we consider that $g \neq 0$, that means there are some sources or sinks of traffic, the equation takes more general form:

$$\frac{\partial q}{\partial x} + \frac{\partial k}{\partial t} = g(t, x), \tag{2.19}$$

where q is flow, k is concentration or density and g represents dissipation (sources and sinks). Equation (2.19) is the final one used for a single–lane continuum-car model.

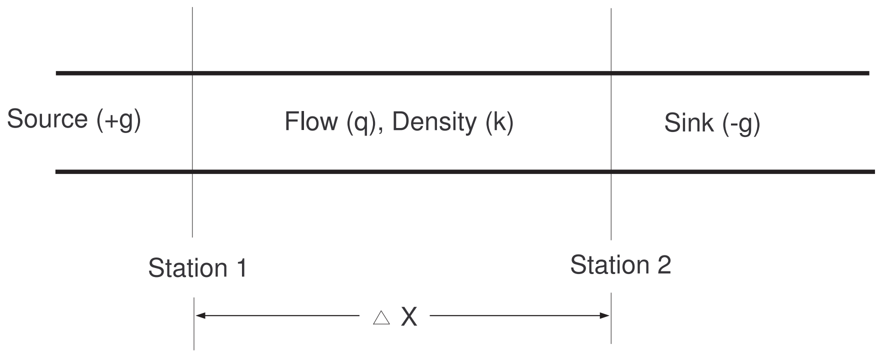

Figure 2.2: Road Section used for Deriving the Conservation Equation

A simple continuum model for desrcibing the flow along two or more lanes can be obtained considering conservation equation of each lane. However, it is more complicated because we must consider the exchange of flow between lanes, which represents the generation or loss of cars in the lane under consideration. The exchange of vehicles between two neighbouring lanes is proportional to the difference of the deviations of their densities from equilibrium values (Gazis, D.C. et al., 1963). These values are well–known lane constants which can be obtained experimentally. Based on these considerations, the following system descibes a flow on a two–lane highway:

$$\frac{\partial q_1}{\partial x} + \frac{\partial k_1}{\partial t} = Q_1$$

$$\frac{\partial q_2}{\partial x} + \frac{\partial k_2}{\partial t} = Q_2$$

where t and x are time and space coordinates, $Q_i(x,t)$ is the lane changing rate, q_i is the flow rate of the i-th lane, and k_i is density of the i-th lane. From these assumptions stated above

$$Q_1 = \alpha\left[(k_2 - k_1) - (k_{20} - k_{10})\right] \tag{2.20}$$

$$Q_2 = \alpha\left[(k_1 - k_2) - (k_{10} - k_{20})\right] \tag{2.21}$$

where α is a sensitivity coefficient describing the intensity of interraction, having units of time^{-1}. Since the system is conserved, it can be easily seen that Q_1+Q_2=0.

Equations (2.20) and (2.21) do not take into account generation or loss of cars that will be introduced at slip lanes. Moreover, when densities k are equal, the changing of the lanes will occur only if $k_{10} \neq k_{20}$. This formula has one defect – even at very low densities, there will be lane changing, and this does not correspond to the real world. This can be rectified by assuming that α is not a constant, but depends on the difference in density between lanes. Time lag, sinks and sources included follow these equations:

$$\frac{\partial q_1}{\partial x} + \frac{\partial k_1}{\partial t} = Q_1 + g(x,t) \tag{2.22}$$

$$\frac{\partial q_2}{\partial x} + \frac{\partial k_2}{\partial t} = Q_2 \tag{2.23}$$

where $g(x,t)$ is the generation rate in lane 1; at off-ramp ramps g is negative. Q_i is lane changing rate in i-th lane:

$$Q_1 = \alpha\left[k_2(x,t-\tau) - k_1(x,t-\tau) - (k_{20} - k_{10})\right]$$
$$Q_2 = \alpha\left[k_1(x,t-\tau) - k_2(x,t-\tau) - (k_{10} - k_{20})\right]$$

where τ is the interaction time lag [s]. In this formula, it is assumed that cars are generated in or departed from line 1, because it is the right lane of the highway. Equations (2.22) and (2.23) can be solved numerically by discretising in time and space (Kuhne, R. and Michalopoulos, P., 2002) and of course generalised to an arbitrary number of lanes into a general form (Ferkl, L., 2007):

$$\frac{\partial k}{\partial t} + \frac{\partial k u_x}{\partial x} + \frac{\partial k u_y}{\partial y} = g(x,y,t), \tag{2.24}$$

where u_x and u_y are the x and y components of the speed vector.

2.1.3 Macroscopic Model

As many cities grew rapidly at the beginning of the 20^{th} century, the important issue of solving the traffic in a city and urban areas was introduced. There is no shortage of techniques to improve traffic flow (traffic signal timing optimization with elaborate computer-based routines as well as simpler, manual, heuristic methods etc.), however, the difficulty lies in evaluating the effectiveness of these techniques. A number of these methods can effectively evaluate the performance of an individual intersection. But a problem arises when these individual components, connected to form the traffic network, are dealt with collectively (Williams, C.J., 2002). Microscopic analyses run into two major difficulties when applied to a street network:

- Each street block or intersection are modelled individually. A proper accounting of the interactions between contiguous network components leads to significant problems.
- Since the analysis is performed for each network component, it is difficult to summarize the results in order to evaluate the overall network performance.

The first problem can be resolved by simulation, but the second one remains. The performance of a traffic system is the response of that system to given travel demand levels. This system consists of the network topology (street width and configuration) and the traffic control system (e.g., traffic signals, designation of one- and two-way streets, and lane configuration). All of the system measurements coming up from the traffic flow theory provide three basic variables of traffic flow: speed, flow, and concentration[4]. These three variables, appropriately defined, can also be used to describe traffic at the network level. This description must be such that it can overcome the intractabilities of existing flow theories when network component interactions are taken into account. Most of the recent approaches are just summations of effects at individual intersections. Travel time models, general network models or **two–fluid** models for instance belong among the best–known models.

2.1.3.1 Travel Time Models

Travel time contour maps provide information about the street network; contours of equal travel time provide information on the average travel times and mean speeds over the network. However, the information is limited because the travel times are related to a single point, and the study can not be used in general and it would likely have to be repeated for other locations. Furthermore, it is demanding to capture network performance with one variable only (e.g., travel time or speed), as the network can be

[4] Some authors use term density.

offering quite different levels of service at the same speed.

Several authors introduced models that can estimate average network travel time or speed as a function of distance from the **central business district** of a city, **CBD**, in opposite to travel time contour maps which consider only travel times away from a specific point.

(VAUGHAN, R. et al., 1972) selected the general model forms providing the best fit to the data from four English cities. They showed that traffic intensity I, defined as the total distance travelled per unit area, tends to decrease with increasing distance from the CBD as follows:

$$I = Ae^{-\sqrt{\frac{r}{a}}}, \tag{2.25}$$

where r is the distance from the CBD, and A and a are parameters. They showed that A and a were unique for each of the cities, while A was also found to vary between peak and off-peak periods. A similar relation was found between the fraction of the area, which is a major road f, and the distance from the CBD,

$$f = Be^{-\sqrt{\frac{r}{b}}}, \tag{2.26}$$

where b and B are parameters for each town.

Several researches were based upon the idea mentioned above. Five different equations relating average speed v to the distance from the CBD r as a result of these researches were introduced. City centres were defined as the point where the radial streets intersected. Average speed for each route section was found by dividing the section length by the actual travel time (kilometres/minute). Constants estimated for data are a, b, and c. A power curve,

$$v = ar^b \tag{2.27}$$

predicts a zero speed in the city centre, that means at r = 0.
Accordingly, (BRANSTON, D.M., 1974) fitted a more general form,

$$v = c + ar^b, \tag{2.28}$$

where c represents the speed at the city centre. There was also a form

$$v = a + br, \tag{2.29}$$

suggested earlier, and strictly linear, up to some maximum speed at the city edge, which was defined as the point where the average speed reached its maximum. A negative exponential function

$$v = a - be^{cr} \tag{2.30}$$

asymptotically approaches some maximum average speed. The last function, suggested by (Lyman, D.A. and Everall, P.F., 1970), also suggested a finite maximum average speed at the city outskirts.

$$v = \frac{1 + b^2 r^2}{a + cb^2 r^2}, \tag{2.31}$$

After several trials (Lyman, D.A. and Everall, P.F., 1970), two of above mentioned equations were left behind immediately:

- The linear model Equation (2.29) overestimated the average speed in the CBDs, reflecting an inability to predict the rapid rise in average speed with increasing distance from the city centre.
- The modified power curve, Equation (2.28), estimated negative speeds in the city centres, and a zero speed for the aggregated data. The original aim of using this model, to avoid the estimation of a zero journey speed in the city centre, was not achieved.

All three remaining functions realistically predict a levelling–off of average speed at the city outskirts, but only the Equation (2.31) indicates a levelling–off in the CBD. However, the power curve, Equation (2.27), showed an overall better fit than the Lyman-Everall model, and was preferred. Later the negative exponential function, Equation (2.30), was also rejected because of its greater complexity in estimation. Truncating the power function at measured downtown speeds was suggested to overcome its drawback of estimating zero speeds in the city centre.

2.1.3.2 General Network Models

A number of models incorporating performance measures other than speed have been proposed. (Smeed, R.J., 1966) introduced model using **network capacity** N as the number of vehicles per unit time that can enter the city. In general, N depends on the general design of the road network, width of roads, type of intersection control, distribution of destinations, and vehicle mix. The principle variables for towns with similar networks, shapes, types of control, and vehicles are: A, the area of the town; f, the fraction of area devoted to roads; and c, the capacity, expressed in vehicles per unit time per unit width of road. These are related as follows:

$$N = \alpha f c \sqrt{A}, \tag{2.32}$$

where α is constant. Some models have defined specific parameters which intend to quantify the quality of traffic service provided to the users in the network. Two principal models are introduced in this section, the α–**relationship**, and the **two–fluid** theory of town traffic.

Three principal variables for the α–**relationship** model were selected: I, the traffic intensity (here defined as the distance traveled per unit area), R, the road density (the length or area of roads per unit area), and v, the weighted space mean speed:

$$I = \alpha \frac{R}{v}, \tag{2.33}$$

where α is different for each city. Relative values of the variables were calculated by finding the ratio between observed values of I and v/R for each sector and the average value for the entire city.

The physical characteristics of the road network, such as street widths, intersection density etc. were found to have a strong effect on the value of α for each zone in a city. Thus, α may serve as a measure of the combined effects of the network characteristics and traffic performance. However, (Buckley, D.G. and Wardrob, J.G., 1980) have shown that α is strongly related to the space mean speed, and (Ardekani, S.A., 1984), through the use of aerial photographs, has shown that α has a high positive correlation with the network concentration.

Two–fluid theory comes up from (Prigogine, I. and Herman, R., 1971) kinetic theory of traffic flow. It says that two distinct flow regimes can be shown – **individual** and **collective**. Both of these are a function of the vehicle concentration. The basic idea is that when the concentration rises so that the traffic is in the collective flow regime, the flow pattern becomes independent of the will of individual drivers.

The kinetic theory deals with multi–lane traffic and therefore, the two–fluid theory of town traffic was proposed as a description of traffic in the collective flow regime in an urban street network. Vehicles in the traffic stream are divided into two classes:

- moving
- stopped

The stopped vehicles class includes vehicles stopped in the traffic stream, i.e., stopped for traffic signals and stop signs, stopped for vehicles being loaded and unloaded which are blocking a moving lane, stopped for normal congestion, etc., but excludes, and it is important, those out of the traffic stream (e.g., parked cars).

The two–fluid model provides a macroscopic measure of the quality of traffic service in a street network which is independent of concentration. The model is based on two assumptions:

- The average running speed in a street network is proportional to the fraction of vehicles that are moving
- The fractional stop time of a test vehicle circulating in a network is equal to the average fraction of the vehicles stopped during the same period.

The variables used in the two–fluid model represent network–wide averages taken over a given period of time. The first assumption of the two–fluid theory relates the average speed of the moving vehicles, V_r , to the fraction of moving vehicles, f_r, in the following manner:

$$V_r = V_m f_r^n, \tag{2.34}$$

where V_m and n are parameters. V_m is the average maximum running speed, and $n \in \mathbb{R}$ is an indicator of the quality of traffic service in the network. The average speed, V, can be defined as $V_r f_r$, and combining with Equation (2.34) we obtain

$$V = V_m f_r^{n+1}. \tag{2.35}$$

Because f_r+f_s=1, where f_s is fraction of vehicles stopped, Equation (2.35) can be rewritten as

$$V = V_m(1 - f_s)^{n+1}. \tag{2.36}$$

This relation can be expressed not only in average speeds, but also in average travel times:

$$\begin{aligned} T &= \frac{1}{V} \\ T_r &= \frac{1}{V_r} \\ T_m &= \frac{1}{V_m}, \end{aligned}$$

where T represents the average travel time, T_r the running (moving) time, and T_s [5] the

[5] As the stop time per unit distance, T_s, increases for a single value of n, the total trip time also increases. Because $T = T_r + T_s$, the total trip time must increase at least as fast as the stop time. If $n = 0$, T_r is constant (2.40), and trip time would increase at the same rate as the stop time. If $n > 0$, trip time increases at a faster rate than the stop time, meaning that running time is also increasing.

stop time, all per unit distance. T_m [6] is the average minimum trip time per unit distance. The second assumption of the two–fluid model relates the fraction of time (a test vehicle circulating in a network is stopped) to the average fraction of vehicles stopped during the same period

$$f_s = \frac{T_s}{T} \,. \tag{2.37}$$

This relation has been proven analytically and represents the ergodic principle embedded in the model, i.e., the network conditions can be represented by a single vehicle appropriately sampling the network.

Equation (2.36) can be rewritten into the terms of travel time as

$$T = T_m(1 - f_s)^{-(n+1)}. \tag{2.38}$$

Together with (2.37)

$$T = T_m[1 - \frac{T_s}{T}]^{-(n+1)}. \tag{2.39}$$

Considering $T = T_r + T_s$ and isolating T_r.

$$T_r = T_m^{\frac{1}{n+1}} T^{\frac{n}{n+1}} \,, \tag{2.40}$$

The formal two-fluid model formulation, then, is

$$T_s = T - T_m^{\frac{1}{n+1}} T^{\frac{n}{n+1}} \,. \tag{2.41}$$

A number of field studies have indicated that urban street networks can be characterised by the two model parameters, n and T.

Intuitively, n must be greater than zero, since the usual cause for increased stop time is congestion. With congestion at high levels, vehicles when moving travel at a lower speed. In fact, field studies have shown that n varies from 0.8 to 3.0, with a smaller value typically indicating better operating conditions in the network. It means n is a measure of the resistance of the network to degraded operation with increased demand. Higher values of n indicate networks that degrade faster as demand increases. Because the two-fluid parameters reflect how the network responds to changes in demand, they must be measured and evaluated in a network over the entire range of demand conditions (Williams, C.J., 2002).

[6]The parameter T_m is the average minimum trip time per unit distance, and it represents the trip time that might be experienced by an individual vehicle alone in the network with no stops. T_m, then, is a measure of the uncongested speed, and a higher value would indicate a lower speed. T_m has been found to range from 2.4 to 4.8 minutes/kilometre, with smaller values typically representing better operating conditions in the network.

2.2 Extension of Traffic Models

It was already proved that continuum flow and car-following models are equivalent with respect to car number and speed in a road section in one moment (Lee, H.K. et al., 2001).

Using the simple continuum model, a variety of simple traffic flow problems can be reproduced analytically by the method of characteristics citeArticle:LIGHTILL and numerically by finite differences (Kuhne, R. and Michalopoulos, P., 2002). However, since the speed in this model is determined by the equilibrium speed–density relationship (2.17), no fluctuation of speed around the equilibrium values is allowed, the model does not faithfully describe nonequilibrium traffic flow dynamics. Therefore, from the theoretical point of view, the simple continuum model does not adequately describe traffic flow dynamics. In order to overcome these shortcomings, a high-order continuum traffic flow model that includes a dynamics equation in addition to the continuity equation was introduced (Jiang, R. et al., 2001). The dynamics equation is derived from car-following theory:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\upsilon}{kT}\frac{\partial k}{\partial x} + \frac{u_e - u}{T}, \tag{2.42}$$

where T is the relaxation time, and $\upsilon = -0.5\frac{\partial u_e}{\partial k}$ is the anticipation coefficient. The left-hand side of (2.42) represents the acceleration. The first term on the right-hand side is so-called anticipation term, which is a respond to drivers' reactions to traffic conditions in front of them. The second term on the right-hand side represents a relaxation to equilibrium, that is, the deviation from the equilibrium speed–density relationship. This advanced model enables to descibe small disturbances in heavy traffic and also allow fluctuations of speed around the equilibrium speed–density relationship. However, a characteristic speed that is greater than the macroscopic flow speed always exists in this model; it means that the future conditions of a traffic flow will be affected by the traffic conditions behind the flow and this is strongly contradictory with the fundamental principle of the traffic flow – vehicles are anisotropic and respond only to frontal stimuli. Therefore a modified approach was chosen (Jiang, R. et al., 2001):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{u_e - u}{T} + c_o\frac{\partial u}{\partial x}. \tag{2.43}$$

Comparing this model with Equation (2.42) and other high–orders models, one can see that the density gradient is replaced by the speed gradient. This replacement in the new model solves the characteristic speed problem that exists in the previous high-order models and therefore enables satisfaction of the anisotropic property of traffic flow.

Proof: (According to JIANG, R. et al., 2001) To show this, Equations (2.43) and (2.19) can be rewritten as:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}\frac{\partial \mathbf{U}}{\partial x} = \mathbf{E}, \tag{2.44}$$

where

$$\mathbf{U} = \begin{pmatrix} k \\ u \end{pmatrix},$$

$$\mathbf{A} = \begin{pmatrix} u & k \\ 0 & u - c_o \end{pmatrix},$$

$$\mathbf{E} = \begin{pmatrix} g \\ (u_e - u)/T \end{pmatrix}.$$

The eigenvalues, λ of the A matrix are found by setting

$$det|\mathbf{A} - \lambda\mathbf{I}| = 0, \tag{2.45}$$

where I is identity matrix. From (2.45)

$$\begin{vmatrix} u - \lambda & k \\ 0 & u - c_o - \lambda \end{vmatrix} = 0,$$

thus $\lambda_1 = u$, $\lambda_2 = u - c_o$. These are the characteristic speeds for the new model expressed by Equation (2.43). Since $c_0 \geq 0$, it follows that the characteristic speeds dx/dt are always less than or equal to the macroscopic flow speed u.

2.3 Comparison of Models

Macroscopic model was developed due to rapid traffic growth in the developing cities. It is purposly modelled for more complex environs such as streets, city districts, and cities themselves and therefore it is obvious that for far more simple traffic flow, such as higway or tunnels, it is improper and unnecessary.

The car-following model was originally developed to model the motion of vehicles following each other on a single lane without overtaking. It is assumed that a driver responds to the car in front of it through acceleration or deceleration. Therefore, the dynamics of the following car is determined by the speed of the leading car and the following car itself, the distance between the two cars, the road conditions, the capability of the car, etc. The classic car-following theory is represented by Equation (2.8). There had been introduced several advanced car-following models, but none had brought something break–through. Most of them argued that there are two types of theories on regulations of car-following. The first type is based on the assumption that the driver of each vehicle seeks a safe following distance from its leading vehicle. The second type assumes that the driver seeks a safe speed determined by the distance from the leading vehicle. According to (Jiang, R. et al., 2001), in the real world exists a common driver behaviour that none of the existing car-following models can explain. That is, when the distance between two vehicles is shorter than the safe distance, the driver of the following vehicle may not decelerate if the preceding vehicle travels faster than the following vehicle because the headway between the two vehicles will become larger. Taking into account this fact a modification of car-following model was introduced (Jiang, R. et al., 2001):

$$\ddot{x}_f(t+T) = \kappa(\dot{x}_l(t) - \dot{x}_f(t)) + \lambda(\dot{x}_l(t) - \dot{x}_f(t)), \qquad (2.46)$$

where κ is a reaction coefficient and λ is the sensitivity. This model considers the effects of both the distance and the relative speed of two successive vehicles, theoretically it is more realistic and exact than the previous ones.

As mentioned above, the macroscopic model is not suitable for tunnel traffic modelling. It is very complex and demanding model, which takes into account variety of factors. Many of them are not neccessary, some of them are even harm to use for simplier traffic such as in highway tunnel. The continuum flow models are suitable and much better for tunnels simulation, because drivers maintain a constant speed inside a tunnel during normal operation. However, great disadvantage of continuum flow model is its unability to simulate car accidents, represented by a sudden closure of one or more lanes, at arbitrary positions. Therefore, from the practical point of view, the **car-following model** seems to be the best choice for implementation.

Model	**Pros**	**Cons**
Car-following	Simple description Real–time Simulates every vehicle Incorporates vehicle dynamics	Slow for large areas Limited amount of vehicles
Continuum flow	Simple description Easy to extend	Unable to simulate accidents No access to a single vehicle
Macroscopic	Covers wide areas	Too complex High comptutational demands

Table 2.1: Comparison of traffic models.

Chapter 3

Implementation

For the computer implementation, it is necessary to discretise continuous equations. This chapter describes process of discretisation, choice of algorithm and implementation of the chosen algorithm.

3.1 Discretisation of Car-Following Model

In Chapter 2, we chose the car-following model as the most suitable one for a highway tunnel. For the computer implementation, it is necessary to discretise it, because Equation (2.8) is in its continuous form. This equation is a simple version of the car-following model, well–suited for the purposes of simulation of traffic in a highway tunnel. The effects of acceleration and deceleration of the vehicles are neglected, because vehicles typically maintain a constant speed in a tunnel. The proportinality factor λ is considered linear. Therefore (2.8) may be discretised as follows:

- Because of linearity of λ, it is possible to integrate both sides of Equation (2.8). After rewriting the coefficient f as i and l as $i-1$ (the cars are counted from the right side), Equation (2.8) looks like

$$\dot{x}_i(t+T) = \lambda(x_{i-1}(t) - x_i(t)) \tag{3.1}$$

- Thereafter Equation (3.1) is discretised and looks like

$$\frac{x_i(t+T) - x_i(t)}{T} = \lambda(x_{i-1}(t) - x_i(t)) \tag{3.2}$$

and

$$x_i(t+T) = x_i(t) + T\lambda(x_{i-1}(t) - x_i(t)), \tag{3.3}$$

respectively.

Taking into account the maximum construction speed of the vehicle, or a speed limit, v_{max}, the Equation (3.1) may be rewritten as

$$\frac{x_i(t+1) - x_i(t)}{\delta x} = x_i(t) + T\lambda(x_{i-1}(t) - x_i(t)), \text{ if } x_{i-1}(t) - x_i(t) < v_{max} \tag{3.4}$$

$$\frac{x_i(t+1) - x_i(t)}{\delta x} = x_i(t) + v_{max}, \text{ if } v_{max} < x_{i-1}(t) - x_i(t), \tag{3.5}$$

or more compactly as

$$\frac{x_i(t+1) - x_i(t)}{\delta x} = \min\left(\left(v_{max}\frac{\delta t}{\delta x}\right), \left(0.5\frac{(x_{i-1}(t) - x_i(t)) - d_{car}}{\delta x}\right)\right) \tag{3.6}$$

where $x_i(t)$ is ith vehicle position at time t [m], v_{max} is a speed limit $[ms^{-1}]$, d_{car} is vehicle lenght [m] δt is a discretisation of time [s], and δx is a discretisation of length [m]. Evaluating δt=1s and δx=1m, Equation (3.6) can be rewritten as follows:

$$x_i(t+1) = x_i(t) + \min\left((v_{max}, (0.5(x_{i-1}(t) - x_i(t) - d_{car}))\right). \tag{3.7}$$

Equation (3.7) means that every single vehicle tries to follow preceding vehicle in an interval of two seconds (Ferkl, L., 2007). If there is enough space between vehicles, the following vehicle maintains the maximum speed – v_{max}.

3.2 Algorithms

There are several possible approaches to implement traffic algorithms. The most important one is the car–motion algorithm, which includes car–motion itself a,nd the algorithm of overtaking. The generally accepted form of the car–motion algorithm is as follows:

```
For each vehicle, v:
CALL routine DriversMotivation to determine whether this driver is moti-
vated to change lanes; now
        IF so, THEN CALL routine OvertakingLane to identify which
          of neighbouring lanes are acceptable as potential target lanes.
                IF the lane to the right is acceptable, THEN
                         CALL routine HasFreeSpace to determine whether a lane-
change is feasible, now.
                         Set flag if so.
                ENDIF
                IF the lane to the left is acceptable, THEN
                         CALL routine HasFreeSpace to determine whether a lane-
change is feasible, now.
                         Set flag, if so.
                ENDIF
                IF both lane-change flags are set (lane-
change is feasible in either direction), THEN
                         CALL routine ChooseBetterLane to determine more favorable target lane
                ELSE IF
                         one lane-change flag is set, THEN
                         Identify that lane
                ENDIF
                         IF a target lane exists, THEN
                         CALL routine ChangeTheLine to execute the lane-change.
                         Update lane-change statistics
                ELSE
                         CALL routine CarFollowing to move vehicle within this lane.
                         Set vehicle's process code to indicate vehi-
cle has been moved this time-step.
                ENDIF
                ELSE
                         CALL routine CarFollowoing to move vehicle within its current lane
                         Set vehicle's process code.
                ENDIF
```

The above algorithm is disclosed in Appendix C as a flow diagram.

3.3 Specific Features of the Traffic Simulator

The Traffic Simulator *(TS)* was developed with aim for maximum user's convinience and easy-to-use control e.g., drag-and-drop components etc. *TS* enables simulation in real–time; it means hours and days can be simulated in minutes and hours. There are four types of cars used for simulation – passenger with petrol engine, passenger with diesel engine, vans and lorries. Each car has its own construction limits – construction speed and other specifications unique for each vehicle such as length, width etc. The entire highway can be modelled from user blocks. A user can defined its length and other proporties e.g. possibility of overtaking (left, right, both, none). The *TS* has automatic checking of the correctness of composed highway; it means only continuous higways are allowed etc. On the other hand, there is possibility to compose higway with slip road blocks; the algorithm used in the *TS*, car-following, enables to simulate closure of one lane (arbitrary number of lanes), or even closure of whole tunnel. The programme is written in such a way that it enables implmentation of air flow simullation (air pollution) in the future in order to become a competitive complex simulator of highway tunnels.

Chapter 4

Conclusion

The main goal of the thesis was a choice of a proper method for implementation of a simulator for highway and higway tunnel traffic. The choice had to take into account several factors; possibility of acceleration and deceleration of vehicles, and multiple lanes being the most important ones. The implementation of the chosen method should enable an easy-to-use and user-friendly environment which led to programming of Traffic Simulator. It is possible to simulate an arbitrary number of lanes; four types of vehicles, setting several options (road length, car–flow for each lane, etc.,) with possibility to extend this model into a future tunnel closure, lane closure, ventilation and emission monitoring, etc. It should enable a complex simmulation of traffic either on a highway or in a highway tunnel. Moreover, it will provide a strong tool for control management of tunnel to train the tunnel operators even before the tunnel is built.

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Appendix A

Content of Accompanied CD

There is an accompanied CD-ROM with the thesis, and implemented Traffic Simulator in MS Visual Studion 2005 C# .

- Folder 1 : Bachelor Thesis
- Folder 2 : Traffic Simulator

Appendix B

Screenshots of the Traffic Simulator

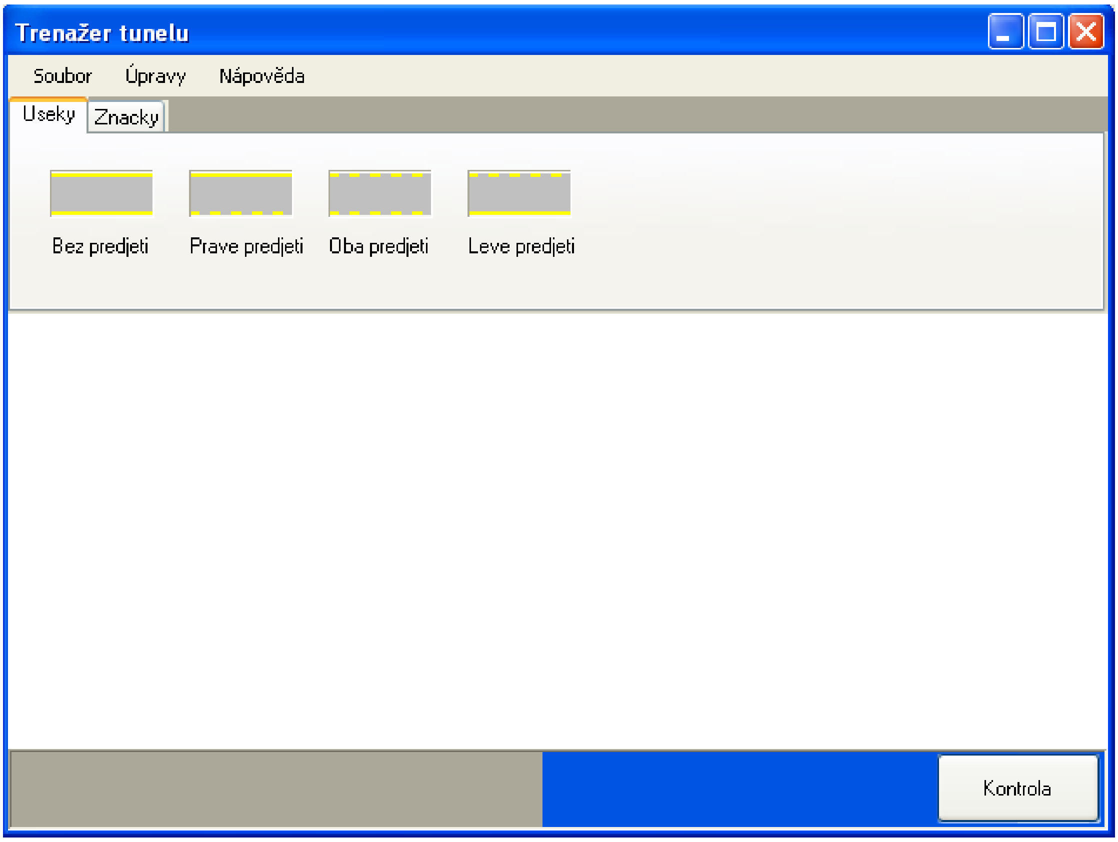

Figure B.1: Simulation main window.

A user can model highway from arbitrary number of user blocks. There are for types of blocks to choose from – overtaking allowed from right side, from left side, from both sides and overtaking forbidden.

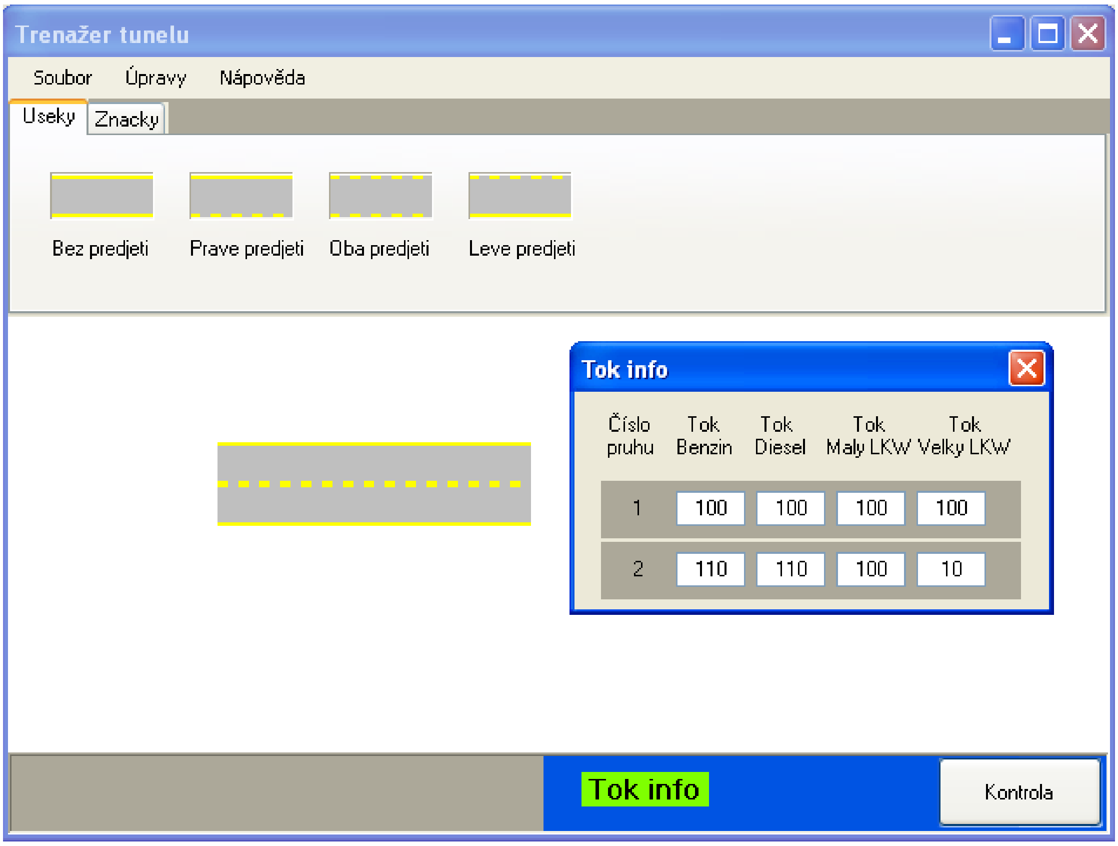

Figure B.2: window for setting flows of car for each lane.

A user can define the number of cars generated per unit time (hour) for each lane. Negative or zero values are not allowed. There are for types of cars to choose from – passenger with petrol engine, passenger with diesel engine, vans and lorries.

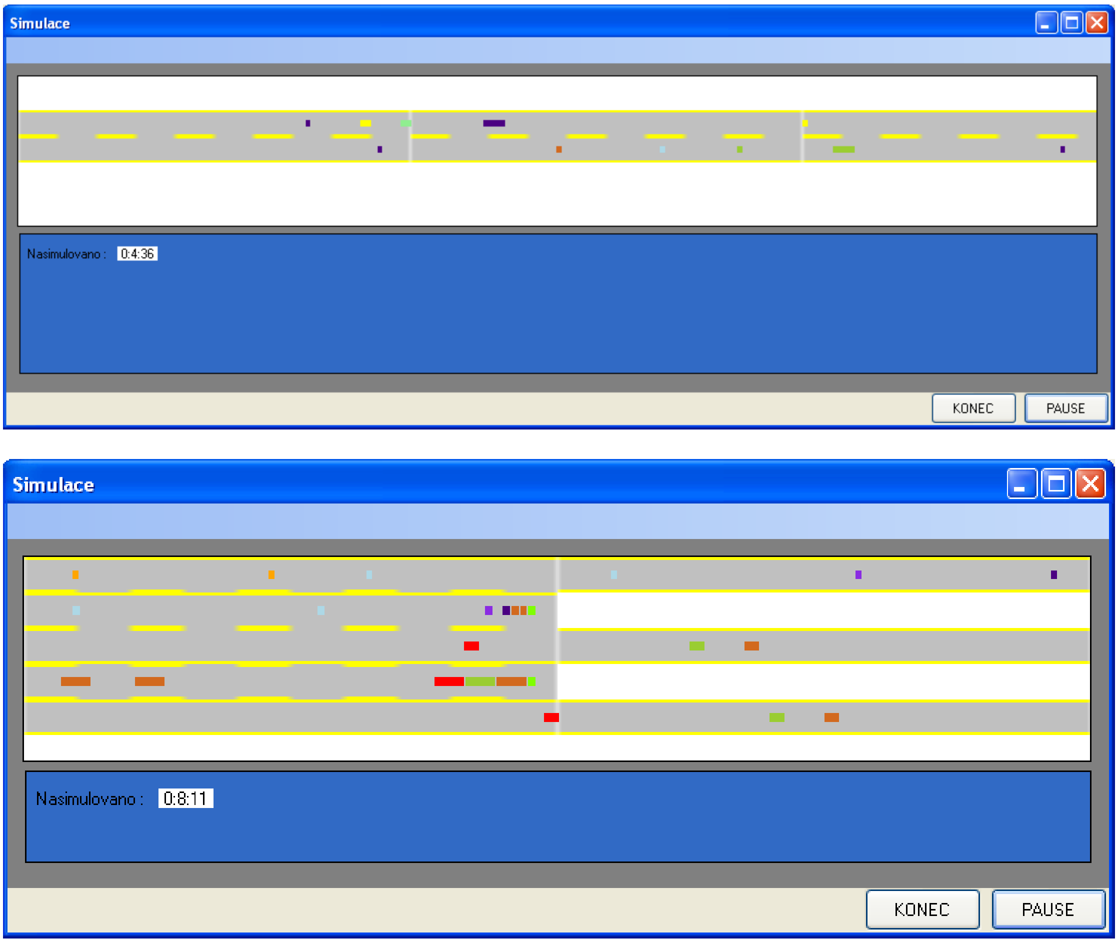

Figure B.3: Simulation in process.

The simulation itself can be paused and resumed. Figures above show two cases of simulation – normal operation and closure of some of the lanes.

Appendix C

Flow Diagram of the Traffic Algorithm