

report on the PhD Dissertation  
**Scheduling under energy consumption limits**

by István Módos

Czech Technical University in Prague –Faculty of Electrical Engineering

Christian Artigues

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## Overview



The manuscript written by István Módos considers a scheduling problem with energy constraints, inspired from situations occurring in high energy-consuming manufacturing companies such as steel hardening and tempered glass production. Namely, a set of jobs has to be scheduled on dedicated machines. The goal is to find a schedule that minimizes the makespan such that jobs assigned to the same machine do not overlap and a maximum energy requirement is satisfied. To evaluate the energy requirement, the time horizon is decomposed in consecutive intervals of equal length, called metering intervals. Each job consumes energy inside each metering interval proportionally to the length of the intersection between the execution interval of the job and the metering interval. For each metering interval, the total energy requirement of the jobs that overlap this interval is summed up and the resulting total energy consumption cannot exceed the prescribed maximum energy usage. In addition to the deterministic setting, a stochastic variant of the problem, where each job may deviated from its assigned start time, is considered. Such a deviation may cause more jobs to overlap inside a metering interval and to exceed the maximum energy limit. The structure of the manuscript is then as follows.

Chapter 1 is the general introduction. A detailed review of electricity bill components for manufacturing companies makes the reader understand why the considered problem is of high industrial and environmental interest. The concept of maximum energy limits in metering intervals is then explained and illustrated. Related work on scheduling energy limitations and robust scheduling are discussed. The main contributions of the thesis are announced.

Chapter 2 gives the scheduling problem statement, introduces the main notations (which are very rigorous but sometimes a bit heavy) and justifies some assumptions : integer start times and dedicated machine environment. Note that precedence constraints between jobs assigned to different machines are not considered while they would be necessary to model classical job-shop environments.

Chapter 3, entitled “Preliminaries” introduces the technical background that was used in the thesis. More precisely, the chapter explains how integer programming, constraint programming, and local search are tailored to scheduling problems. An adaptative local search framework is adopted, where the heuristics adaptively selects the best promising neighborhood in a set of candidate neighborhoods.

Chapter 4 presents the contributions of the thesis to the deterministic scheduling problem. The chapter first focuses on a particular case of the problem with a single machine and identical maximum energy limits on all metering intervals. The problem is shown to be  $\mathcal{NP}$ -complete

by reduction from 3-PARTITION. Then, another particular case with one machine, unit processing time and two metering intervals with the same energy limit, is considered and shown to be  $\mathcal{NP}$ -complete by reduction from PARTITION. A variant with a fixed ordering of the jobs and the additional constraint that each job must start and end within a single metering interval is considered. The problem is shown to be polynomially solvable for a fixed number of machines by shortest path computation. For the most general problem, several exact and heuristic approaches are proposed. A constraint programming (CP) formulation is based on disjunctive resource global constraints for the parallel machine part and overlapping constraints for the energy part. Mixed-integer linear programming (MILP) formulations are proposed, inspired from the literature: a disjunctive MILP, a time indexed MILP and an implicit MILP. For the latter, the start times of the jobs need not be modeled, only the job/interval overlapping variables and binary variables for relative job-interval positioning are present. It is shown that the constraints that ensure that at most one job crosses each interval on each machine are not necessary for large jobs. Contiguity constraints are also designed, which can be simplified into type 2 special ordered sets for small jobs. Another improvement is to avoid big-M constraints for makespan minimization through iterative solving of the MILP, varying the number of intervals. Finally an adaptive local search method is proposed, based on several components. The scheduling operator is a heuristic that solves the problem given a total order on the jobs. It is based on the determination of an earliest start time taking account of the maximum energy constraint. An initial solution is computed by combining priority rules in a total of 24 configurations. Experiments are based on a set of randomly generated instances. There are from 15 to 350 jobs per machine and from 2 to 10 machines. The length of the metering interval is either 15 or 60 minutes, while the job durations and the energy requirements tightness are parameterized. For small instances and within a 10-minute CPU time, the iterative implicit MILP and the time-indexed MILP obtained the best results among MILP approaches. CP and local search methods appear to be the best ones for finding good quality solutions. The results are well detailed and tempered in function of the parameters values. For large instances, the local search approach clearly outperforms all other approaches. This chapter contains solid contributions and the algorithms and computational experiments are well detailed and illustrated. We miss a few details on the implementation of the CP formulation inside CP Optimizer: for instance, to model the maximum energy constraint inside each interval, is the constraint summing up the overlaps directly declared as is ? Or does one use cumul functions linked to auxiliary interval variables representing the overlaps ? Also, as the CP approach is still (slowly) improving the criterion (Fig 4.5) at the end of the time limit, what happens if we allow a larger CPU time, e.g. one hour ?

Chapter 5 presents the second part of the contributions, related to taking uncertainty into account. As already stated, the scenarios consider an increase of the start time of the baseline schedule up to a given limit per job. Given integer start times, the number of scenarios is equal to the product of the upper limits. For the general parallel machine problem a CP model is introduced, defining a sequence per possible scenario, which yields an intractable model. The rest of the chapter is devoted to the single-machine case. For a fixed job sequence and a general regular objective function, a pseudo-polynomial algorithm is proposed. The algorithm is based on the notion of latest start time given a fixed baseline schedule : this is the start time obtained when all deviations are set to their maximum values. From this definition, the realized right shifted schedule is the schedule obtained from the latest start schedule by enforcing non-overlapping constraints. Properties on latest start and right shift schedules are established. Namely, if a baseline schedule is globally earlier than another one, the same holds for their latest start schedule. Also the right shift schedule of a baseline schedule is a realized schedule. The concept of robust baseline schedule is then defined as a baseline schedule such that no realization of the deviations yields a violation of the maximum energy constraint. Using an argument based on the properties of latest start and right shift schedule, the earliest robust baseline schedule is proved optimal. An exponential algorithm is first given to find the earliest robust baseline schedule of a given job permutation, or prove that such a schedule does not exist. Dominance rules allow to decrease the complexity

of the algorithm: the violation checks need only be made on the right shift schedule and, in addition, maximum possible overlaps of jobs with metering intervals are defined, which allows to reduce the number of scenarios to be considered. Overall a cubic complexity is obtained, with an additional multiplicative factor equal to the maximum deviation. From this pseudo-polynomial algorithm, a logic-based benders decomposition is carried out. A master problem (time-indexed MILP) only partially takes maximum deviations into account. Then, the subproblem takes the obtained permutation and compute its earliest robust baseline schedule or proves that there are none. Combinatorial benders cuts are then generated based on the concept of cutting intervals, with the idea to provide cuts stronger than simply forbidding the current master schedule. A simple branch-and-bound approach is devised, enumerating the permutations while making use of the pseudo-polynomial algorithm for upper bound computations. The same algorithm is also used as the scheduling operator of the adaptive local search procedure. Experimental results show that the best methods are the branch-and-bound method that finds the larger number of optimal solutions for small instances and the adaptive local search method that finds the larger number of best solutions, both for small and large instances. In summary, the chapter exploits intensively the pseudo-polynomial algorithm for the fixed-sequence one machine problem, which indeed is a strong results and the major contribution of the chapter.



### Evaluation and recommendation

To conclude, István Módos brings several strong contributions in this PhD thesis, both experimental and theoretical, to the parallel and single machine scheduling problem with makespan minimization and energy consumption constraints ensured on metering intervals. In the deterministic setting, by identifying non trivial polynomial cases and providing NP-hardness results, he contributed strongly to the complexity cartography of this family of problems. He also significantly improved MILP-based formulations proposed for this family of problems in the literature and proposed efficient constructive and local search-based heuristics. As a follow-up, this would be now interesting to establish the link with the complexity results, constructive heuristics and MILP formulations proposed in [1,2] for a related problem considering also metering intervals and makespan criterion without machine constraints, but with precedence relations. To make the link, it would be interesting to see if the proposed methods could be adapted to precedence constraints, as they would be in this case able to solve both problems. István Módos also presents significant contributions to the case with uncertainty, in a robust setting. As already mentioned the pseudo-polynomial algorithm is a major contribution. We may think of a logic-based benders contribution that could be adapted to obtain a tractable variant of the CP model, with a reduced number of scenarios and a sub-problem that could use the same pseudo-polynomial algorithm and issue different combinatorial benders cuts to express the possible need of changing the machine assignment. It would also be appropriate to consider other type of disruptions in addition to the start time deviations: what happens if the duration of the jobs also changes ? However, the already obtained results are already impressive and this is also shown by the publications issued from this thesis: 3 publications in renown journals of the discipline (Computers &OR, Computers & Industrial Engineering, Constraints) plus two submissions (CIE and EJOR) and several international conferences among which a best paper award in CPAIOR.

For all this reasons I undoubtedly recommend the grade "Pass".

### Additional references

- [1] Pierre-Antoine Morin, Christian Artigues, Alain Haït. Periodically Aggregated Resource Constrained Project Scheduling Problem ». In : European Journal of Industrial Engineering 11.6,

p. 792–817, 2017.

[2] Pierre-Antoine Morin. Planification et ordonnancement de projets sous contraintes de ressources complexes, Université de Toulouse 3 Paul-Sabatier, Thèse de Doctorat, 2018

