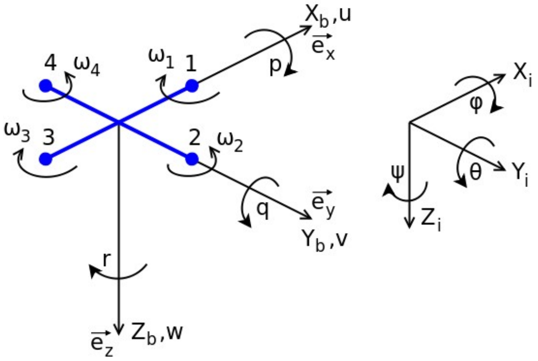
KONTAKT 2011

Modeling, Identification and Control of a Quadrotor Aircraft

Author: Marcelo De Lellis (mardelellis@gmail.com)
Supervisor: Ing. Zdeněk Hurák, Ph.D. (hurak@fel.cvut.cz)

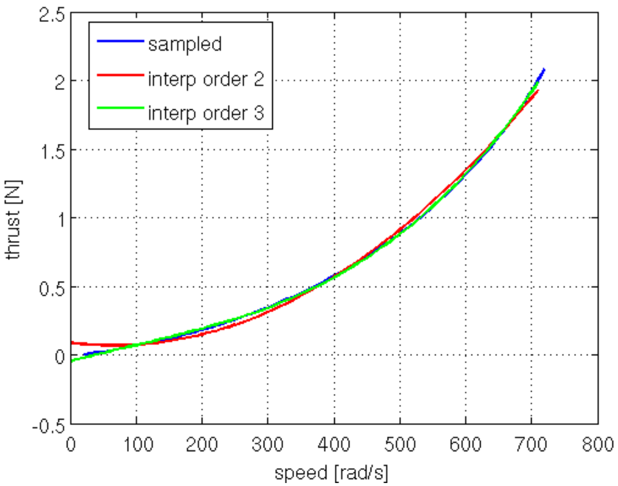
Mathematical Modeling

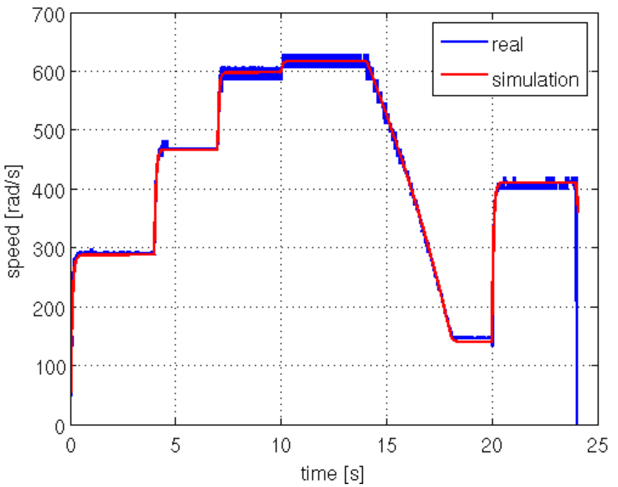
- Moment and force equations
- 4 modeled effects: gravity, gyroscopic moments, thrust, air drag torque on rotors
- Simplification for around hovering condition: no drag force/moments on airframe
- Non-linear and linearized model



Experimental Identification

- Rotor torque experiment for air drag and inertia coefficients
- Thrust experiment
- Weighting for masses, calculation of inertias
- Interpolation of time responses for indirect identification of other parameters





Control Design

- 4 DOFs: (X,Y,Z) + heading
- Classical nested loops (P,PI)
- One-level LQ state-feedback
- For model and parametric uncertainties:
  - I_x, I_y: mixed-sensitivity H_∞
  - I_G: μ-synthesis with DK-iterations

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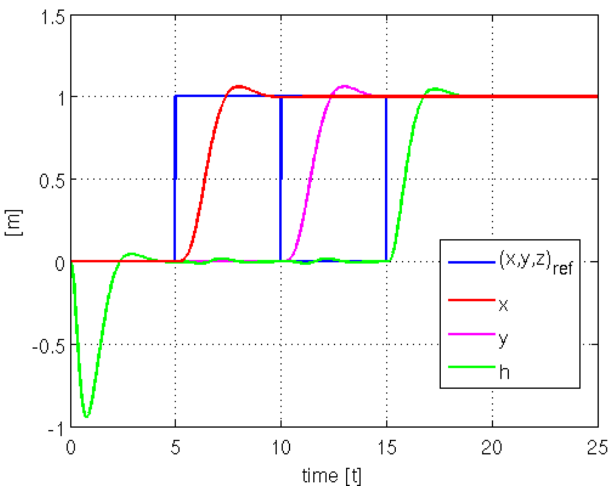

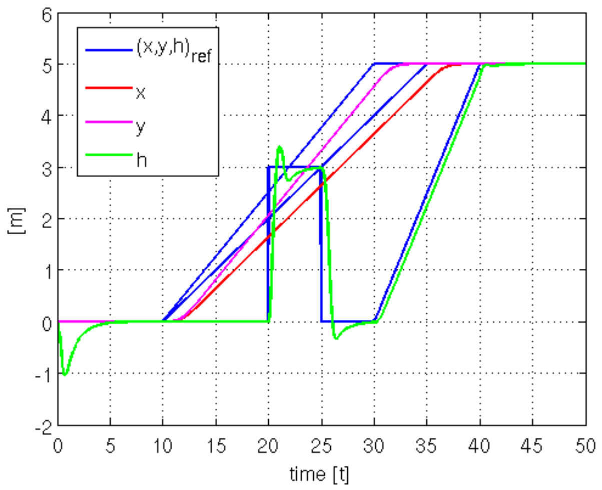

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1) Mathematical Modeling:

- 4 sources of force/moments: Earth's gravity, thrust, gyroscopic moments and air drag torque;
- Assumption: near-hovering condition, hence no air drag on airframe.

$$\dot{p} = \frac{l_a}{I_x}\sum_{i=1}^{3}\gamma_i(\omega_2^i-\omega_4^i)+\frac{I_G}{I_x}q\sum_{j=1}^{4}\omega_j(-1)^j+\frac{(I_y-I_z)}{I_x}q\,r$$

$$\dot{q} = \frac{l_a}{I_x}\sum_{i=1}^{3}\gamma_i(\omega_3^i-\omega_1^i)-\frac{I_G}{I_y}p\sum_{j=1}^{4}\omega_j(-1)^j-\frac{(I_x-I_z)}{I_y}p\,r$$

$$\dot{r} = \frac{1}{I_z} = \sum_{j=1}^{4}(I_G\dot{\omega}_j+k_D\omega_j^2+B_a\omega_j)(-1)^j$$

Eq. 1: Moment equations.

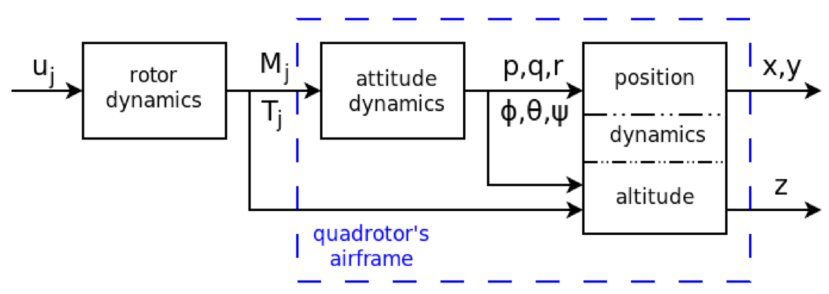

Fig. 1: Quadrotor's dynamic subsystems.

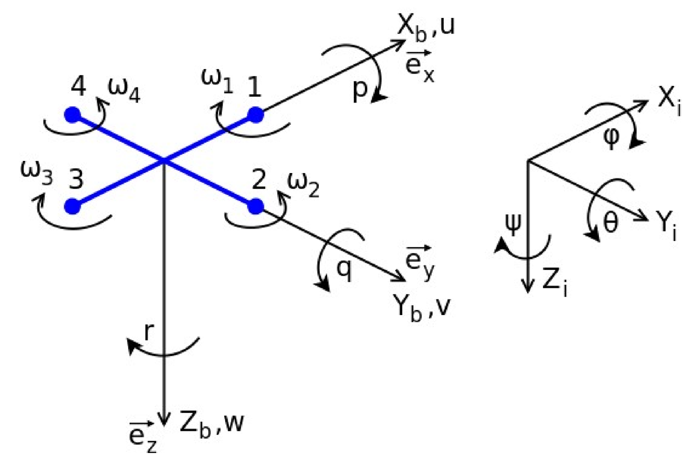

Fig. 2: Coordinate systems.

Fig. 3: Real quadrotor system.

The *moment equations* produce the angular rates which determine the attitude, whereas the *force equations* yield the linear speeds which determine the 3D-position.

$$\dot{u} = v\,r-w\,q-g\,\sin\theta$$

$$\dot{v} = w\,p-u\,r+g\,\sin\phi\,\cos\theta$$

$$\dot{w} = u\,q-v\,p+g\,\cos\phi\,\cos\theta-\frac{1}{m}\sum_{j=1}^{4}\sum_{i=1}^{3}\gamma_i\,\omega_j^i$$

Eq. 2: Force equations.

2) Experimental Identification:

- Measurement of rotor torque and thrust;
- Weighting of masses and analytical determination of moments of inertia;
- Step response matching and interpolation for assessment of other parameters.

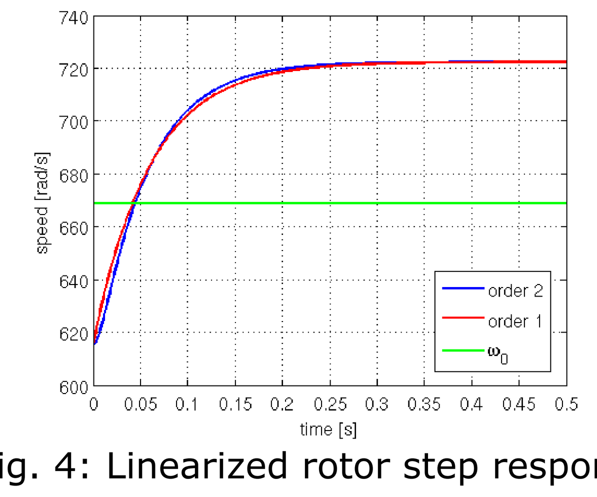

Fig. 4: Linearized rotor step response

Linearized rotor 2nd-order transfer function can be approximated by a 1st-order model.

$$G_{R_s}(s) = \frac{K_s}{s+\lambda_s} = \frac{885.6}{s+16.7}$$

Eq. 3: Rotor simplified dynamics.

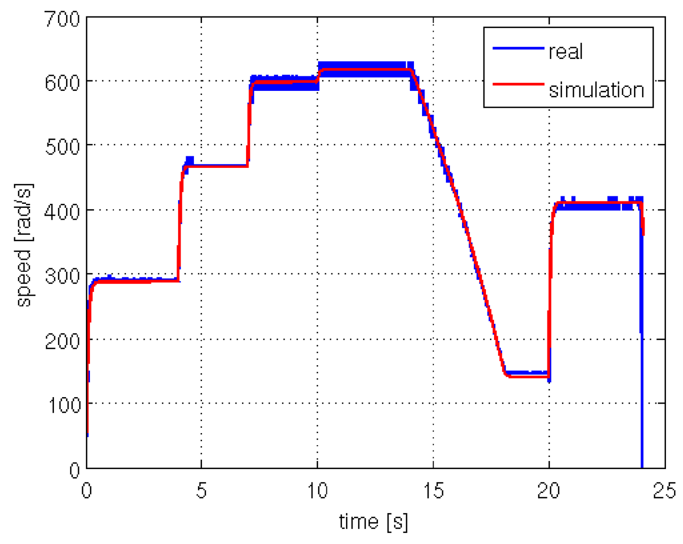

Fig. 5: Rotor simulation against real data for model validation.

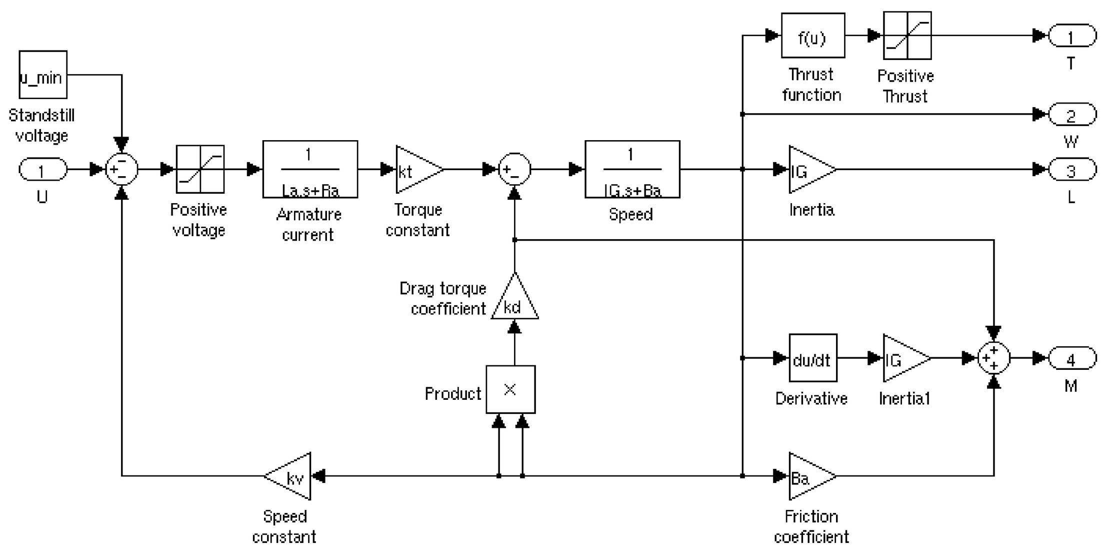

Fig. 6: Simulink implementation of rotor.

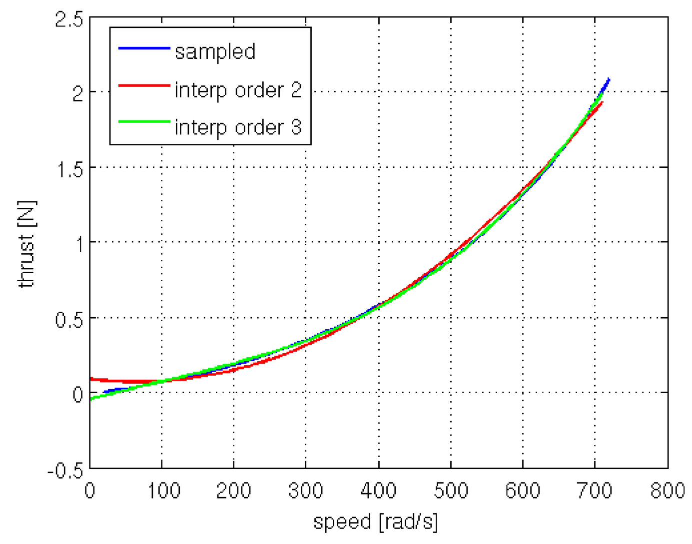

Fig. 7: Approximations for thrust behavior.

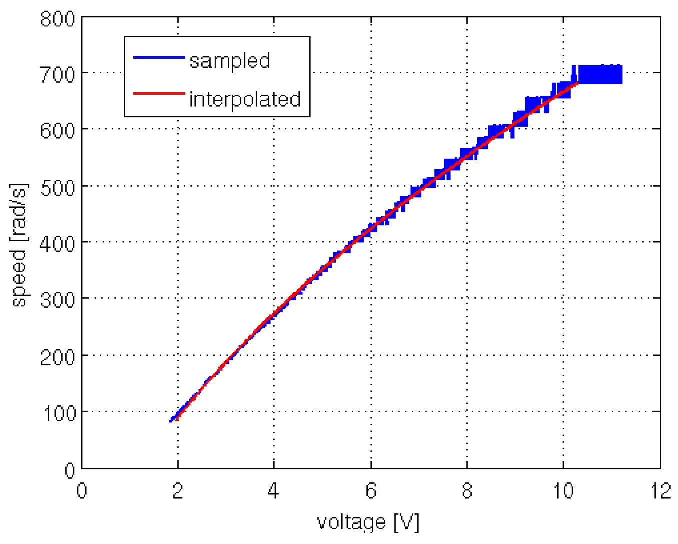

Fig. 8: Effect of non-linear air drag damping on rotor gain $u \to \omega$.

3) Control Design:

- Goals: reference tracking, disturbance rejection and robustness to model and parametric uncertainties;
- 4 approaches: classical (P,PI), LQ state-feedback, mixed-sensitivity (MH) H_∞ and μ-synthesis with DK-iterations.

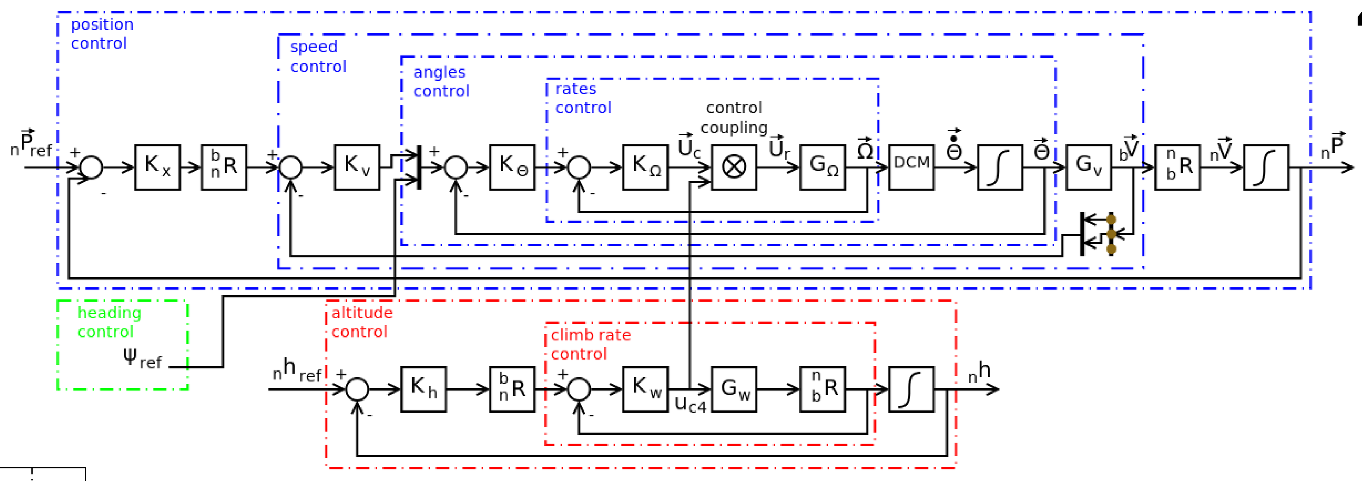

Fig. 9: Classical control with hierarchical architecture.

Considering robustness, MS H_∞ produced better results in terms of performance (oscillations and settling time). μ-syn DK-iterations yielded oscillations, although less, even for nominal case. Control action presented some saturation.

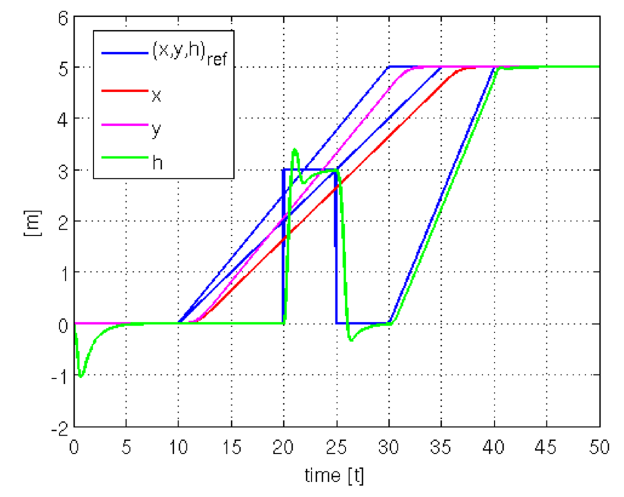

Fig. 11: Performance of K_w designed with MS- H_∞ for $I'_x=I'_y=4I_{x0}$.

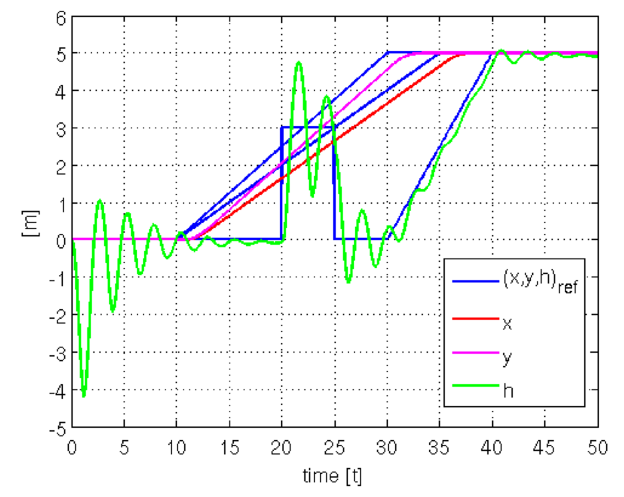

Fig. 12: Performance of K_w designed with μ-syn DK-iterations for $I_G=3.1I_{G0}$.

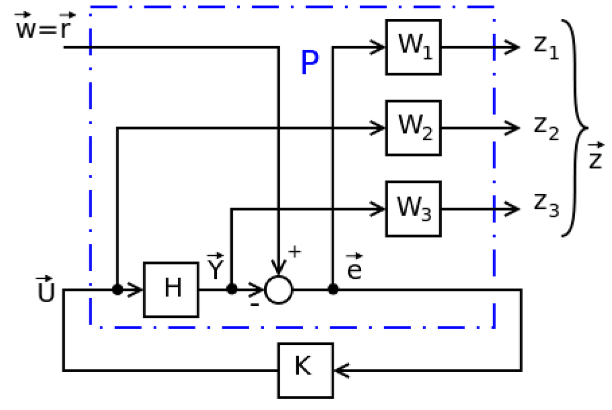

Fig. 13: Block diagram for design of mixed-sensitivity H_∞ controller.

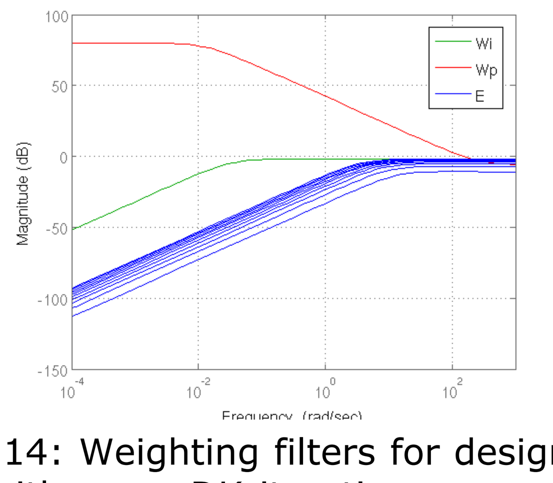

Fig. 14: Weighting filters for design of K_w with μ-syn DK-iteration approach.

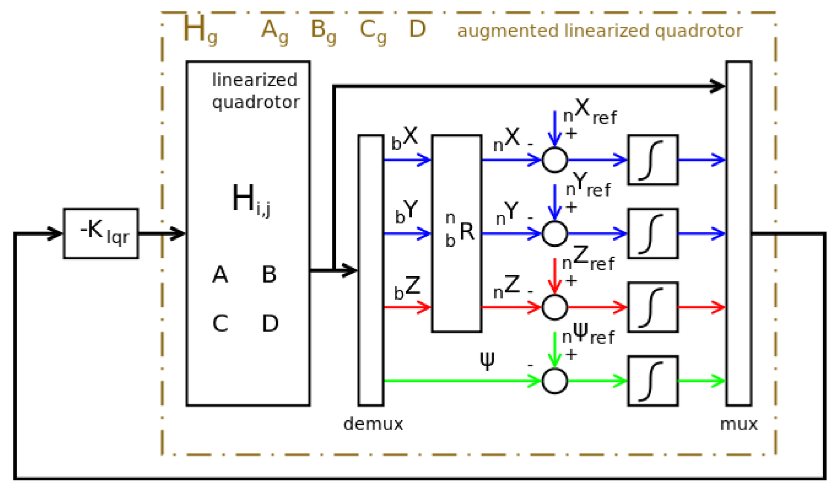

Fig. 15: LQ control structure.

4) Kalman Filtering:

- Linear approach works well for ψ because it does not depend on linearization point.

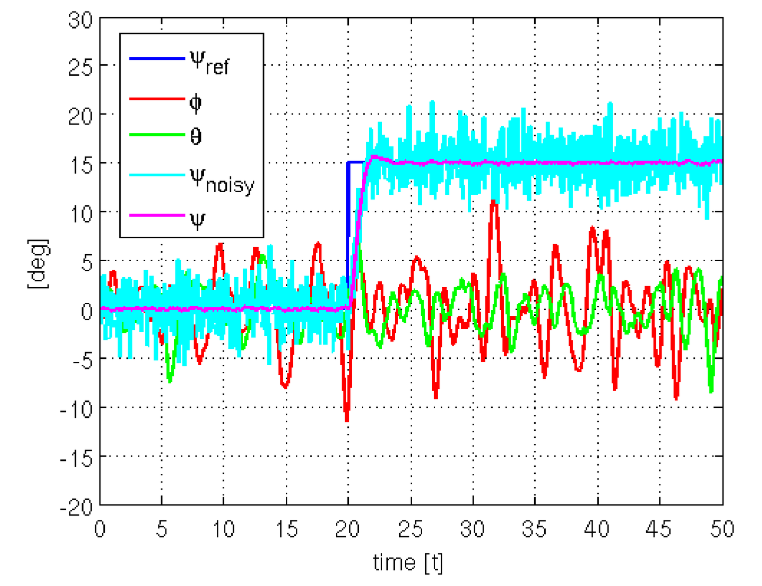

Fig. 10: Kalman filter results on angles. Oscillations due to reaction of feedback control to noise.

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