

## Review of PhD. thesis

Title: Numerical Algorithms of Quadratic Programming for Model Predictive Control

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Reviewer: Ing. David Horák, PhD.

### a) Up-to-dateness of the chosen topic:

It is nice to see an effort to optimize quadratic programming (QP) algorithms not for powerful supercomputers, but for processors with low performance such as automotive control units. This is very interesting for me and Mr. Šantin has done a huge amount of work. He studied the basic approaches, implemented viable algorithms in MATLAB, systematically examined their behaviour and impact of various parameters on model benchmarks and demonstrated their efficiency on real world problems such as linear control of the light-duty diesel engine air path or nonlinear control of the heavy-duty diesel engine air path, in both cases with the purpose to reduce the formation of nitrogen oxides – especially this is very hot topic nowadays when Dieselgate affair culminates.

### b) Fulfilment of stated goals:

Mr. Šantin has developed three main solvers (CGNP, PND, NPP) for embedded application of MPC in Chapters 5-7. These algorithms are characterized by low number of the required iterations,  $O(n^2)$  computational complexity and  $n^2$  floating point numbers memory requirements. Their convergence is independent on QP problem Hessian conditioning because of the second-order information in the solution of the auxiliary face problem defined by the current active set. The ingredients preserving good scalability are expansion and proportioning steps modifying the active set, so that the QP problem size and initial approximation do not affect their behaviour. Numerical experiments with real world problems in Chapter 8 mentioned above compare these three developed methods with those in available software (qpOASEs, FORCES, FiOrdOs). All the algorithms are described precisely including their

convergence rates, complexities and the reached results are sufficiently commented. The best performance was observed in case of NPP algorithm.

**c) Thesis layout and used methodology:**

It is obvious, that the author has thought over the thesis structure very carefully. Its chapters and sections follow each other logically, so the thesis is nicely readable. The references are chosen properly and reflect state-of-the-art. An excellent English language enhances the final impression and gives an opportunity to present easily the reached results. As an applied mathematician, I appreciate very much a nice "story" beginning with the specification of embedded MPC application, introducing newly designed algorithms including their convergence and complexity analysis and finishing with very nice applications of diesel engine air path controllers. Great!

I suggest several changes: Page 2: relies -> rely, Page 16: might be removed -> might be removed, Page 25 and 29: Than -> Then, Page 41: Lemma 5.2.2 - there exist -> there exists, Page 95: PND solvers shows -> show.

**d) New contribution of PhD. thesis:**

As the author summarized, the contributions can be classified as follows: (1) Tutorial results containing an overview of the linear MPC framework, QP problems and existing methods for their solution. (2) Preliminary algorithmic results – m-rank update procedure of Cholesky factors for the face problem solution and efficient cost function gradient updates (e.g. extension of PLS algorithm). (3) Main algorithmic and theoretical results dealing with CGNP, PND and NPP algorithms, their convergence and complexity analysis. The introduced algorithms use PLS routine to modify rapidly the active set by means of the projected-Newton-direction path which results in the low number of iterations and its independency on the initial approximation and size of the Hessian and its conditioning. Implemented proportionality test in PND and NPP methods reduces the number of unwanted active sets changes. Proportioning step in the CGNP and PND algorithms results in the efficient active set reduction with fixed step-size, which is further improved in NPP algorithm. (4) Comparison of the developed and implemented algorithms to the state-of-the-art methods on several benchmarks.

**e) Importance of the reached results for industry and science:**

As already mentioned in previous items, three developed algorithms are highly-valued as they beat other state-of-the-art methods. Author was successful to manage the convergence and complexity analysis and demonstrated their applicability and efficiency not only for model problems, but also for engineering problems of diesel engine air path controllers. The presented algorithms with author's crucial ingredients converged in at most 15-20 low complexity iterations independently of the initial iterate, QP problem size and conditioning.

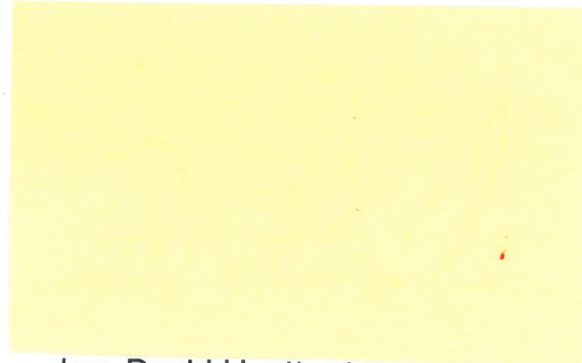
**f) Fulfilment of the self-creative scientific work and published results:**

Author has introduced systematically all his inventions and commented them properly in this thesis. Many of these valuable results have been presented at conferences, Mr. Šantin is author or co-author of 1 article in an international journal with impact factor, 1 in a reviewed international journal, 2 patents and 9 international conference papers.

**Conclusion:**

The PhD. thesis is with no doubts of very high-quality and the amount of author's done work is exceptionally extensive.  
I recommend this thesis to its defence.

In Ostrava, 17.11.2016



Ing. David Horák, PhD.